z-Scaling of cumulative hadron production in pA collisions at high energies

A. Aparin, M. Tokarev
JINR, Dubna, Russia
Contents

- Introduction
- $z$-Scaling (ideas, definitions, properties, ...)
- Self-similarity of high-$p_T$ pion production in $pp$ & $pA$ collisions, $\sqrt{s}=11.5-38.8$ GeV
- Self-similarity of low-$p_T$ cumulative pion production in $pA$ ($A=\text{Li, Be, C, Al, Cu, Ta}$)
- Conclusions
Motivation & Goals

Search for possible signatures of new physics phenomena in inclusive pp & pA collisions

Analysis of experimental data on inclusive spectra of hadron production in pA collisions to verify properties of z-scaling in low-p_T cumulative region

- pA is a reference frame for pp & AA
- cumulative process:
  - enhancement of nuclear matter compression
  - particle formation is sensitive to state of matter
  - search for indications of phase transition & CP
Phase diagram of strongly interacting matter

The phase diagram of water is established

- Phases (ice I-XV, liquid, vapor)
- Phase boundaries
- Phase transitions
- Triple Point (16)
- Critical Point (2)

The phase diagram of strongly interacting matter is under study

- Phases - ?
- Phase boundaries - ?
- Phase transitions - ?
- Triple Point - ?
- Critical Point - ?
Motivation of using $z$-scaling

$z$-scaling can be used as a tool to search for new physics in particle production in pp, AA & pA at high energies.

$z$-scaling reveals self-similar properties in hadron, jet and direct photon production in high energy hadron and nucleus collisions.

Description of hadron spectra using $z$-scaling approach
Development of $z$-scaling theory as a tool of physics analysis
Scaling & Universality

$\pi^-, K^-, \bar{p}, \Lambda$ in pp collisions

- Energy & angular independence
- Flavor independence ($\pi, K, p, \Lambda$)
- Saturation for $z<0.1$
- Power law for high $z>4$

Energy scan of spectra at U70, ISR, S$p\bar{p}$S, SPS, HERA, FNAL (fixed target), Tevatron, RHIC, LHC

M.T. & I.Zborovsky
T.Dedovich
J.Mod.Phys.3, 815 (2012)

Scaling – “collapse” of data points onto a single curve.
Scaled particle yield ($\Psi$) vs. scaled transverse momentum ($z$).
Universality classes – hadron species ($\varepsilon_F, \alpha_F$).
**z-Scaling**

**Principles: locality, self-similarity, fractality**

**Locality:** collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).

**Self-similarity:** interactions of the constituents are mutually similar.

**Fractality:** the self-similarity over a wide scale range.

**Hypothesis of z-scaling:**

Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

\[ s^{1/2}, p_T, \theta_{\text{cms}} \]

\[ x_1, x_2, \delta_1, \delta_2 \]

\[ \frac{E d^3 \sigma}{dp^3} \] Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable \( z \).
Locality of hadron interactions

M.T. & I.Zborovský
Yu.Panebratsev, G.Skoro
JINR E2-99-113

Constituent subprocess

\[(x_1 M_1) + (x_2 M_2) \Rightarrow (m_1) + (x_1 M_1 + x_2 M_2 + m_2)\]

Kinematical condition (4-momentum conservation law):

\[(x_1 P_1 + x_2 P_2 - p)^2 = M_X^2\]

Recoil mass: \[M_X = x_1 M_1 + x_2 M_2 + m_2\]
Self-similar parameter $z$

$$z = z_0 \Omega^{-1}$$

$$z_0 = \frac{\sqrt{s_{\perp}}}{(dN_{\text{ch}}/d\eta|_0)m}$$

- $\sqrt{s_{\perp}}$ is the transverse kinetic energy of the subprocess consumed on production of $m_1$ & $m_2$
- $dN_{\text{ch}}/d\eta|_0$ is the multiplicity density of charged particles at $\eta = 0$
- $m$ is an arbitrary constant (fixed at the value of nucleon mass)
- $\Omega^{-1}$ is the minimal resolution at which a constituent subprocess can be singled out of the inclusive reaction.
Fractal measure $z$

The fractality is reflected in definition of $z$

$$z = z_0 \Omega^{-1}$$

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}$$

$\Omega$ is relative number of configurations containing a sub-process with fractions $x_1, x_2$ of the corresponding 4-momenta

$\delta_1, \delta_2$ are parameters characterizing structure of the colliding objects

$\Omega^{-1} (x_1, x_2)$ characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

$$z(\Omega) \bigg|_{\Omega^{-1} \to \infty} \to \infty$$

The fractal measure $z$ diverges as the resolution $\Omega^{-1}$ increases.
Scaling function $\Psi(z)$

$$\int_0^\infty \Psi(z)dz = 1$$

$z \rightarrow \alpha_F z, \quad \Psi \rightarrow \alpha_F^{-1} \Psi$

$$\Psi(z) = \frac{\pi}{(dN/d\eta) \cdot \sigma_{\text{inel}}} \cdot J^{-1} \cdot E \frac{d^3\sigma}{dp^3} \quad \longleftrightarrow \quad \int E \frac{d^3\sigma}{dp^3} dy d^2p_\perp = \sigma_{\text{inel}} \cdot N$$

- $\sigma_{\text{in}}$ - inelastic cross section
- $N$ - average multiplicity of the corresponding hadron species
- $dN/d\eta$ - pseudorapidity multiplicity density at angle $\theta$ ($\eta$)
- $J(z,\eta;p_T^2,y)$ - Jacobian
- $E d^3\sigma/dp^3$ - inclusive cross section

The scaling function $\Psi(z)$ is probability density to produce an inclusive particle with the corresponding $z$. 

---

Hadron structure, June 30 – July 4, 2013
A-dependence of $z$-scaling

The scaling transformations of $z$ and $\Psi(z)$ allow us to compare scaling functions for different nuclei.

$$z \rightarrow \alpha(A) \cdot z$$
$$\Psi(z) \rightarrow \alpha^{-1}(A) \cdot \Psi(z)$$

$$\alpha(A) \approx 0.9 A^{0.15}$$

Self-similarity of nuclear modification of constituent interactions and hadron formation.

$$z = z_0 \, \Omega^{-1}$$
$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}$$

$$\delta_1 = A_1 \delta, \ \delta_2 = A_2 \delta$$

M.T., Yu.Panebratsev, I.Zborovsky, G.Skoro

Hadron structure, June 30 – July 4, 2013
Self-similarity of hadron production in $pp$

Spectra

- 10 orders of magnitude
- Sensitive to energy $\sqrt{s}$ at high $p_T$
- Power law for high $\sqrt{s}$ and $p_T$

High-$p_T$ Spectra

- Energy independence of $\Psi(z)$
- Power law of $\Psi(z)$ at high $z$

Scale invariance

Independence of the shape of the curve on $\{z, \Psi\}$ plane on scale quantities $\sqrt{s}, p_T, \theta$

Self-similarity of hadron production in pA

Strong dependence of spectra on $\sqrt{s}$ at high $p_T$

- Energy independence of $\Psi(z)$
- Power law of $\Psi(z)$ at high $z$
- $A$-dependence of $\Psi(z)$

Scale invariance
Independence of the shape of the curve on $\{z, \Psi\}$ plane on scale quantities $\sqrt{s}, p_T, \theta$


M.T., Yu. Panebratsev, I. Zborovsky, G. Skoro
Cumulative pion spectra in pA at FNAL

- Spectra in cumulative region: \( p > 0.5 \text{ GeV/c} \).
- Smooth behavior of spectra vs. \( p \).
- Stronger angular dependence with \( p \).
- \( A \)-dependence of spectra (\( A=7-181 \)).

Universal shape of $\Psi(z)$

Power law for $z > 4$

No discontinuity of $\delta_2 = A_2 \delta$

Scale invariance

Independence of the shape of the curve on $\{z, \Psi\}$ plane on scale quantities $\sqrt{s}, p_T, \theta$

$z \rightarrow \alpha(A) z$  

$\Psi \rightarrow \alpha^{-1}(A) \Psi$
High-p$_T$ and low-p$_T$ pion production in pA

C, Al & D

- Collapse of data points
- Universal shape of $\Psi(z)$
- Self-similarity over a wide kinematic range

$\theta^\pi_{lab} = 180^\circ$

<table>
<thead>
<tr>
<th>$p$</th>
<th>D</th>
<th>C</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.447</td>
<td>0.905</td>
<td>5.13</td>
<td>10.6</td>
</tr>
<tr>
<td>0.456</td>
<td>0.928</td>
<td>5.53</td>
<td>12.2</td>
</tr>
<tr>
<td>0.459</td>
<td>0.933</td>
<td>5.63</td>
<td>12.7</td>
</tr>
</tbody>
</table>


A. Aparin & M.T. (2013)
High-$p_T$ and low-$p_T$ pion production in $pA$

Cu, Ta & D

- Collapse of data points
- Universal shape of $\Psi(z)$
- Self-similarity of hadron production over a wide range of energy $\sqrt{s}$, angle $\theta$ transverse momentum $p_T$ and atomic number $A$

<table>
<thead>
<tr>
<th>$\theta_{\text{lab}}$</th>
<th>$p_L$ (GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= 180^0$</td>
<td>70</td>
</tr>
<tr>
<td>$p_{\pi_{\text{max}}}$ (GeV/c)</td>
<td>400</td>
</tr>
<tr>
<td>0.447</td>
<td>20.7</td>
</tr>
<tr>
<td>0.905</td>
<td>37.8</td>
</tr>
<tr>
<td>0.456</td>
<td>27.9</td>
</tr>
<tr>
<td>0.928</td>
<td>69.8</td>
</tr>
<tr>
<td>0.459</td>
<td>30.0</td>
</tr>
<tr>
<td>0.933</td>
<td>84.7</td>
</tr>
</tbody>
</table>

A. Aparin & MT (2013)
Momentum fractions $x_1$ & $x_2$ vs. $p_T$

Proton fragmentation region
$0 < x_1 < 1$
Non-cumulative region
$x_2 < 1/A$

Nucleus fragmentation region
$0 < x_2 < 1$
Cumulative region
$x_2 > 1/A$
Cumulative pion spectra in pA

predictions based on $z$-scaling

- Spectra in cumulative region: $p > 0.5 \text{ GeV/c}$
- Smooth behavior of spectra vs. $p_T$
- Verification of the additive law $\delta_A = A\delta$

Self-similarity
High-\(p_T\) and low-\(p_T\) pion production in \(pA\)

**FNAL** (J.Cronin, G.Leksin, D.Jaffe) & **U70** (R.Sulyaev)

- Beam Energy Scan in \(pA\)
- Spectra of cumulative identified particles
- Multiplicity density \(dN_{ch}/d\eta\) vs. \(\sqrt{s}\) and \(\eta\)
- Centrality dependence of spectra
- Power law \(\Psi(z) \sim z^{-\beta}\), in cumulative region
- Discontinuity of fractal dimensions \(\delta_1, \delta_2\)

Suggestions to \(pA\) physics program:

Goal: Search for phase transition & CP  \(\leftrightarrow\)  Search for violation of \(z\)-scaling

Hadron structure, June 30 – July 4, 2013
Conclusions

- The FNAL data on cumulative pion spectra in $pA$ collisions at $\sqrt{s}=27.4$ GeV were analyzed in $z$-scaling approach.
- Results of this analysis were compared with previous ones from the data obtained by J. Cronin, R. Sulyaev and D. Jaffe groups.
- Indication on self-similarity of the pion production in $pA$ collisions at low-$p_T$ in the cumulative region were obtained.
- Universality of the shape of $\Psi(z)$ was used to predict the pion spectra in $pA$ collisions in the deep-cumulative range ($1/A << x_2 < 1$).

$z$-Scaling of hadron production in $pA$ collisions at high energies manifests self-similarity, locality and fractality of hadron interactions at a constituent level.

The results can be used to develop the program to search for new physics phenomena in $pA$ collisions at U70, RHIC, LHC & NICA, FAIR
Thank you for your attention!
Back-up slides
Self-similarity of hadron production in \( pD \)

Spectra
- 10 orders of magnitude
- Sensitive to energy \( \sqrt{s} \) at high \( p_T \)
- Power law for high \( \sqrt{s} \) and \( p_T \)

High-\( p_T \) Spectra

Energy independence of \( \Psi(z) \)
- Power law of \( \Psi(z) \) at high \( z \)

Fractal dimensions in \( pA \) & \( AA \)
\[ \delta_1 = A_1 \delta, \quad \delta_2 = A_2 \delta \]

Cumulative pion spectra in pA

$\sqrt{s}=27.4$ GeV

$A=\text{Li, Be, C, Al, Cu, Ta}$

$\theta_{lab} = 70^0, 90^0, 118^0, 160^0$

Beam Atomic Number and Energy Scan in Cumulative Processes

Hadron structure, June 30 – July 4, 2013
Angular dependence of $dN_{ch}/d\eta (\sqrt{s, \theta, A})$

- The shape of $\Psi(z)$ is the same for all nuclei
- Restoration of normalization of $\Psi(z)$ at fixed $\theta_{lab}$ over a wide range of $p_T$
Momentum fractions $x_1, x_2$

Principle of minimal resolution: The momentum fractions $x_1, x_2$ are determined in a way to minimize the resolution $\Omega^{-1}$ of the fractal measure $z$ with respect to all constituent sub-processes taking into account 4-momentum conservation:

$$\Omega = (1 - x_1)^\delta_1 (1 - x_2)^\delta_2$$

$$\frac{\partial \Omega}{\partial x_1} \bigg|_{x_2 = x_2(x_1)} = 0$$

Momentum conservation law:

$$(x_1 P_1 + x_2 P_2 - p)^2 = M_X^2$$

Recoil mass

$$M_X = x_1 M_1 + x_2 M_2 + m_2$$
Transverse kinetic energy $\sqrt{s_\perp}$

\[
s_{\perp}^{1/2} = (s_{\chi}^{1/2} - M_1 \lambda_1 - M_2 \lambda_2) - m_1 + (s_{\chi}^{1/2} - M_1 \chi_1 - M_2 \chi_2) - m_2
\]

- Energy consumed for the inclusive particle $m_1$
- Energy consumed for the recoil particle $m_2$

**Fraction decomposition:**

\[
x_{1,2} = \lambda_{1,2} + \chi_{1,2}
\]

\[
\lambda_{1,2} = \kappa_{1,2} + \nu_{1,2}
\]

\[
\chi_{1,2} = (\mu_{1,2}^2 + \omega_{1,2}^2)^{1/2} + \omega_{1,2}
\]

\[
\omega_{1,2} = \mu_{1,2} U, \quad U = \frac{\alpha - 1}{2\sqrt{\alpha}} \xi, \quad \alpha = \frac{\delta_2}{\delta_1}
\]

\[
\xi^2 = (\lambda_1 \lambda_2 + \lambda_0) / [(1 - \lambda_1)(1 - \lambda_2)]
\]

**The scaling variable $z$ and scaling function $\Psi(z)$ are expressed via relativistic invariants.**