ADVANCED NUCLEON ELECTROMAGNETIC STRUCTURE MODEL AND CHARGE PROTON RMS RADIUS

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**Motivation**

- Nucleons are non-point-like particles [Hofstadter (1958)].
- Internal electromagnetic (EM) structure of nucleons are described by EM form factors – scalar functions of one variable \( q^2 = t \). Usually the Pauli and Dirac form factors \( F_1, F_2 \) or the Sachs form factors \( G_E, G_M \) are used.
- The EM form factors of nucleons are usually measured
  - in elastic scattering processes \( \ell \, N \rightarrow \ell \, N \) (space-like region \( t < 0 \))
  - in annihilation processes \( \ell^- \, \ell^+ \rightarrow N \bar{N} \) (time-like region \( t > 4m_N^2 \))
The electromagnetic structure of the proton \((G_E^P, G_M^P)\) is well measured on both space-like and time-like region.

- removed space-like data on \(G_E^P\) at high momentum transfer \((t < -1\text{GeV}^2)\) due to low accuracy

Precise polarization measurements of the elmag proton form factors ratio \(\mu_P G_E^P / G_M^P\) in the space-like region

Data on the electromagnetic structure of the neutron also exist in both kinematical regions

The proton charge radius was measured by independent Lamb shift method - muon hydrogen atom
For the description of the nucleon electromagnetic structure we are using the Unitary and Analytic approach developed by S. Dubnička.

Similarly to the well known VMD model it has to be saturated by several vector mesons.

Formerly eight and ten-resonance U&A models of the nucleon were developed, however due to the recent advances in both theory and experiment we have revisited this problem:

- new experimental data on the nucleon elmag form factors (very precise Mainz data, new polarization data, ...)
- new (correct) method of introducing the asymptotic behavior of the electromagnetic form factors
- discovery of new vector meson resonances allows construction of the U&A model conserving SU(3) symmetry
The U&A model is inspired by well known VMD model.

- it utilizes nonlinear transformation

\[ t = t_0 - \frac{4(t_{in} - t_0)}{[1/W(t) - W(t)]^2} \]

and introduce the width of the vector mesons to fulfill unitary and analytic properties of the elmag form factors

To conserve the SU(3) symmetry the same number of the \( \rho, \omega, \phi \) vector mesons and theirs excitations are used to saturate the model.
In order to describe asymptotic behavior given by the quark model \((F_2^\nu|_{t \to \infty} \sim t^{-3})\) we need at least 9 resonances.

Such minimal SU(3) symmetric U&A model has 12 free parameters \((t_1^s, t_1^v, t_2^s, t_2^v, a_1, .., a_8)\).

General form of the parametrization

\[
FF = \left( \frac{1 - W^2(t)}{1 - W^2_N} \right)^{2n} \times Reson(t_1^s, t_1^v, t_2^s, t_2^v, a_1, .., a_8)
\]

asymptotic \(\sim t^{-n}\) resonance behavior
9-RESONANCE U&A MODEL OF THE NUCLEON

\[
F_1^s = \left( \frac{1 - V(t)^2}{1 - V_N^2} \right)^4 \left[ \left( \frac{1}{2} - \sum_{v=\omega,\omega',\phi,\phi'} a_v \right) LH(V_{\omega''})LH(V_{\phi''}) + \right.
\]
\[
+ LH(V_{\omega}) \left( LH(V_{\phi''}) \frac{C(V_{\phi''}) - C(V_{\omega})}{C(V_{\phi''}) - C(V_{\omega''})} + LH(V_{\omega''}) \frac{C(V_{\omega''}) - C(V_{\omega})}{C(V_{\omega''}) - C(V_{\phi''})} \right) a_{\omega}^{(1)}
\]
\[
+ LH(V_{\omega'}) \left( LH(V_{\phi''}) \frac{C(V_{\phi''}) - C(V_{\omega'})}{C(V_{\phi''}) - C(V_{\omega''})} + LH(V_{\omega''}) \frac{C(V_{\omega''}) - C(V_{\omega'})}{C(V_{\omega''}) - C(V_{\phi''})} \right) a_{\omega'}^{(1)}
\]
\[
+ LH(V_{\phi}) \left( LH(V_{\phi''}) \frac{C(V_{\phi''}) - C(V_{\phi})}{C(V_{\phi''}) - C(V_{\omega''})} + LH(V_{\omega''}) \frac{C(V_{\omega''}) - C(V_{\phi})}{C(V_{\omega''}) - C(V_{\phi''})} \right) a_{\phi}^{(1)}
\]
\[
+ LH(V_{\phi'}) \left( LH(V_{\phi''}) \frac{C(V_{\phi''}) - C(V_{\phi'})}{C(V_{\phi''}) - C(V_{\omega''})} + LH(V_{\omega''}) \frac{C(V_{\omega''}) - C(V_{\phi'})}{C(V_{\omega''}) - C(V_{\phi''})} \right) a_{\phi'}^{(1)}
\]
9-resonance U&A model of the nucleon

\[
F_1^\nu = \left( \frac{1 - W(t)^2}{1 - W^2_N} \right)^4 \left[ \left( \frac{1}{2} - a_{\rho} \right) LH(W_{\rho'}) LH(W_{\rho''}) \right.
+ LH(W_{\rho}) \left( LH(W_{\rho'}) \frac{C(W_{\rho'}) - C(W_{\rho})}{C(W_{\rho'}) - C(W_{\rho''})} + LH(W_{\rho''}) \frac{C(W_{\rho''}) - C(W_{\rho})}{C(W_{\rho''}) - C(W_{\rho'})} \right) a_{\rho}^{(1)} \left. \right]
\]

\[
F_2^\nu = \frac{1}{2} (\mu_p - \mu_n - 1) \left( \frac{1 - X(t)^2}{1 - X^2_N} \right)^6 LH(X_{\rho}) LH(X_{\rho'}) LH(X_{\rho''})
\]
\[ F_s^2 = \left( \frac{1 - U(t)^2}{1 - U_N^2} \right)^6 \left[ \left( \frac{\mu_p + \mu_n - 1}{2} - \sum_{\nu = \omega, \phi, \omega'} a_\nu \right) \text{LH}(U_\omega') \text{LH}(U_\phi') \text{LH}(U_\phi'') \right] + \text{LH}(U_\omega) \left( \text{LH}(U_\phi') \text{LH}(U_\phi'') \right) \frac{C(U_\phi') - C(U_\omega)}{C(U_\phi') - C(U_\omega'')} \frac{C(U_\phi'') - C(U_\omega)}{C(U_\phi'') - C(U_\omega')} 
+ \text{LH}(U_\omega'') \text{LH}(U_\phi'') \frac{C(U_\omega'') - C(U_\omega)}{C(U_\omega'') - C(U_\phi')} \frac{C(U_\phi'') - C(U_\omega)}{C(U_\phi'') - C(U_\omega')} 
+ \text{LH}(U_\omega'') \text{LH}(U_\phi') \frac{C(U_\omega'') - C(U_\omega)}{C(U_\omega'') - C(U_\phi'')} \frac{C(U_\phi') - C(U_\omega)}{C(U_\phi'') - C(U_\phi')} 
+ \text{LH}(U_\phi) \left( \text{LH}(U_\phi') \text{LH}(U_\phi'') \right) \frac{C(U_\phi') - C(U_\phi)}{C(U_\phi') - C(U_\omega'')} \frac{C(U_\phi'') - C(U_\phi)}{C(U_\phi'') - C(U_\omega')} 
+ \text{LH}(U_\omega'') \text{LH}(U_\phi'') \frac{C(U_\omega'') - C(U_\phi)}{C(U_\omega'') - C(U_\phi'')} \frac{C(U_\phi'') - C(U_\phi)}{C(U_\phi'') - C(U_\phi')} \]
9-resonance U&A model of the nucleon

\[
\begin{align*}
+ \quad & LH(U_{\omega''}) LH(U_{\phi'}) \left( \frac{C(U_{\omega''}) - C(U_{\phi})}{C(U_{\omega''}) - C(U_{\phi''})} - \frac{C(U_{\phi'}) - C(U_{\phi})}{C(U_{\phi'}) - C(U_{\phi''})} \right) a^{(2)}_{\phi'} \\
+ \quad & LH(U_{\omega'}) \left( \frac{C(U_{\phi'}) - C(U_{\omega'})}{C(U_{\phi'}) - C(U_{\omega''})} - \frac{C(U_{\phi''}) - C(U_{\omega'})}{C(U_{\phi''}) - C(U_{\omega''})} \right) \frac{C(U_{\omega''}) - C(U_{\phi'})}{C(U_{\phi'}) - C(U_{\omega''})} - \frac{C(U_{\phi''}) - C(U_{\omega'})}{C(U_{\phi''}) - C(U_{\omega''})} \frac{C(U_{\phi'}) - C(U_{\phi})}{C(U_{\phi'}) - C(U_{\phi''})} a^{(2)}_{\omega'} \\
+ \quad & LH(U_{\omega''}) LH(U_{\phi''}) \left( \frac{C(U_{\omega''}) - C(U_{\phi})}{C(U_{\omega''}) - C(U_{\phi''})} - \frac{C(U_{\phi'}) - C(U_{\phi})}{C(U_{\phi'}) - C(U_{\phi''})} \right) \frac{C(U_{\omega''}) - C(U_{\phi'})}{C(U_{\phi'}) - C(U_{\omega''})} - \frac{C(U_{\phi''}) - C(U_{\omega'})}{C(U_{\phi''}) - C(U_{\omega''})} \frac{C(U_{\phi'}) - C(U_{\phi})}{C(U_{\phi'}) - C(U_{\phi''})} \\
+ \quad & LH(U_{\omega''}) LH(U_{\phi'}) \left( \frac{C(U_{\omega''}) - C(U_{\phi})}{C(U_{\omega''}) - C(U_{\phi''})} - \frac{C(U_{\phi'}) - C(U_{\phi})}{C(U_{\phi'}) - C(U_{\phi''})} \right) \frac{C(U_{\omega''}) - C(U_{\phi'})}{C(U_{\phi'}) - C(U_{\omega''})} - \frac{C(U_{\phi''}) - C(U_{\omega'})}{C(U_{\phi''}) - C(U_{\omega''})} \frac{C(U_{\phi'}) - C(U_{\phi})}{C(U_{\phi'}) - C(U_{\phi''})} a^{(2)}_{\omega'} 
\end{align*}
\]
RESULTS - DATA POINTS

- In our analysis we have considered 353 experimental points on nucleon form factors with addition of the recent very precise Mainz data points (1-2 orders of magnitude smaller errors) on proton electromagnetic form factors.

- We have used 77 Mainz data points on proton form factors calculated by Rosenbluth separation and for comparison 1189 Mainz experimental points on the differential cross section of the electron scattering on the proton.
RESULTS - $G_E^P, G_M^P$
RESULTS - $G_E^N, G_M^N$
RESULTS - RATIOS: $\mu_P G_E^P / G_M^P, \mu_N G_E^N / G_M^N$
<table>
<thead>
<tr>
<th></th>
<th>N. of exp.points</th>
<th>$\chi^2 / N_{df}$</th>
<th>$\sqrt{\langle r_p^2 \rangle}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosenbluth</td>
<td>430</td>
<td>2.04</td>
<td>$0.863 \pm 0.007$ fm</td>
</tr>
<tr>
<td>Diff. cross. section</td>
<td>1542</td>
<td>1.76</td>
<td>$0.858 \pm 0.007$ fm</td>
</tr>
<tr>
<td>w/out Mainz data</td>
<td>353</td>
<td>1.87</td>
<td>$0.864 \pm 0.006$ fm</td>
</tr>
</tbody>
</table>
CONCLUSIONS

- It is possible to describe all relevant experimental data on the proton and nucleon electromagnetic form factor in both SL and TL kinematical regions simultaneously with the proposed 9-resonance U&A model.

- The inclusion of the new very precise Mainz experimental data on the proton elmag structure didn’t change the behavior of the form factors and the proton radius significantly, therefore we assume they are in good agreement with the older data.

- However Mainz Rosenbluth data generate higher $\chi^2$ which may suggest problem with the Rosenbluth separation or underestimated errors.

- Contrary to the our older analysis with the 10-resonance U&A model, proton charge radius is in disagreement with the Lamb shift measurement on the muon hydrogen atom ($\sim 0.84$ fm) – reopened puzzle.
RESULTS - $G^P_E, G^P_M$