R-broken SUSY Standard Model with right-handed neutrino

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MSSM + neutrino Yukawa interactions
MSSM + neutrino Yukawa interactions + R-parity breaking
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<td>$V^k$</td>
<td>Weak $W^k$ ($W^\pm, Z$)</td>
<td>wino, zino $\tilde{W}^k$ ($\tilde{W}^\pm, \tilde{Z}$)</td>
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<td>Hypercharge $B$ ($\gamma$)</td>
<td>bino $\tilde{b} (\tilde{\gamma})$</td>
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### Matter

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<tr>
<td>$L_i$</td>
<td>sneutrons $\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$</td>
<td>leptons $L_i = (\nu, e)_L$</td>
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<td>2</td>
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<td>$E_i$</td>
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<td>squarks $\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$</td>
<td>quarks $Q_i = (u, d)_L$</td>
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<td>2</td>
<td>$1/3$</td>
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<td>$U_i$</td>
<td>$\tilde{U}_i = \tilde{u}_R$</td>
<td>quarks $U_i = u_R$</td>
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<td>$D_i$</td>
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### Higgses

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<th>Higgsinos $\tilde{H}_1, \tilde{H}_2 (\tilde{h}_1, \tilde{h}_2, \tilde{h}^\pm)$</th>
<th>$SU_b(3)$</th>
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<tr>
<td>$H_1$</td>
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<td>$(h, H, A, H^\pm)$</td>
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<td>$H_2$</td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
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</tbody>
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Higgs bosons in the MSSM

- At the tree level the MSSM Higgs potential has the form

\[
V_{\text{tree}}(H_1, H_2) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + h.c.) + \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{g_2^2}{2} |H_1^+ H_2|^2
\]

Note: the Higgs self-interaction coupling constant is fixed and is determined by the gauge interactions, this case differs from the Standard Model.

- The MSSM Higgs potential is positively defined and has no non-trivial non-zero minimum.
Higgs bosons in the MSSM

- Running of the Higgs masses leads to the phenomena known as radiative electroweak symmetry breaking.

\[
V_{\text{tree}}(H_1, H_2) = m_1^2 |H_1|^2 - |m_2|^2 |H_2|^2 - m_3^2 (H_1 H_2 + h.c.) + \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2
\]

- One obtains conditions for the electroweak symmetry
Higgs bosons in the MSSM

- The physical spectrum of the MSSM Higgs sector consists of 5 states:

  \[ G^0 = -\cos \beta P_1 + \sin \beta P_2 \quad \text{Goldstone boson} \rightarrow Z_0 \]
  \[ A = \sin \beta P_1 + \cos \beta P_2 \quad \text{Neutral CP} = -1 \text{ Higgs} \]
  \[ G^+ = -\cos \beta (H_1^-) + \sin \beta H_2^+ \quad \text{Goldstone boson} \rightarrow W^+ \]
  \[ H^+ = \sin \beta (H_1^-) + \cos \beta H_2^+ \quad \text{Charged Higgs} \]
  \[ h = -\sin \alpha S_1 + \cos \alpha S_2 \quad \text{SM Higgs boson} \ CP = 1 \]
  \[ H = \cos \alpha S_1 + \sin \alpha S_2 \quad \text{Extra heavy Higgs boson} \]

- Compare to the Standard Model with 1 Higgs boson.
Higgs bosons in the MSSM

- One can calculate the Higgs masses diagonalising the corresponding mass matrices.

- Masses of the CP-odd and charged Higgs bosons

  \[ m_A^2 = m_1^2 + m_2^2 \]

  \[ m_{H^\pm}^2 = m_A^2 + M_W^2 \]

- Masses of the CP-even Higgs bosons

  \[ m_{h,H}^2 = \frac{1}{2} \left[ m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2M_Z^2\cos^2 2\beta} \right] \]

- If \( m_A \gg M_Z \), the lightest Higgs boson is lighter than the Z-boson!

  \[ m_h \approx M_Z |\cos 2\beta| < M_Z \]
Higgs bosons in the MSSM

- The inequality \( m_h \approx M_Z |\cos 2\beta| < M_Z \) is spoiled by radiative corrections

\[
m_h^2 \approx M_Z^2 \cos^2 2\beta \\
+ \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{m_t^2 m_t^2}{m_t^4} \\
+ 2 \text{ loops}
\]

- 1-loop contribution is very large and positive
- 2-loop contribution is much smaller and negative
Supersymmetric Standard Model

Superpotential

\[ \mathcal{W}_R = y^{ij}_u \bar{u}_i Q_j \cdot H_u - y^{ij}_d \bar{d}_i Q_j \cdot H_d - y^{ij}_e \bar{e}_i L_j \cdot H_d + \mu H_u \cdot H_d, \]

SUSY breaking

\[ - \frac{1}{2} (M_3 \tilde{g}^\alpha \tilde{g}^\alpha + M_2 \tilde{W}^\alpha \tilde{W}^\alpha + M_1 \tilde{B} \tilde{B} + \text{h.c.}), \]
\[ - m^2_{Qij} \tilde{Q}_i^\dagger \cdot \tilde{Q}_j - m^2_{uij} \tilde{u}_i^\dagger \tilde{u}_j - m^2_{dij} \tilde{d}_i^\dagger \tilde{d}_j, \]
\[ - m^2_{Lij} \tilde{L}_i^\dagger \cdot \tilde{L}_j - m^2_{eij} \tilde{e}_i^\dagger \tilde{e}_j. \]
\[ - m^2_{H_u} H_u^\dagger \cdot H_u - m^2_{H_d} H_d^\dagger \cdot H_d - (b H_u \cdot H_d + \text{h.c.}) \]
\[ - a^{ij}_u \tilde{u}_i \tilde{Q}_j \cdot H_u + a^{ij}_d \tilde{d}_i \tilde{Q}_j \cdot H_d + a^{ij}_e \tilde{e}_i \tilde{L}_j \cdot H_d + \text{h.c.} \]
Supersymmetric Standard Model

Superpotential (R-parity conserving)

\[ W_R = y_{ij}^u \bar{u}_i Q_j \cdot H_u - y_{ij}^d \bar{d}_i Q_j \cdot H_d - y_{ij}^e \bar{e}_i L_j \cdot H_d + \mu H_u \cdot H_d, \]

Superpotential (R-parity breaking)

\[ W_{\Delta L=1} = \lambda^{ijk}_e L_i \cdot L_j \bar{e}_k + \lambda^{ijk}_L L_i \cdot Q_j \bar{d}_k + \mu^i L_i \cdot H_u, \]
\[ W_{\Delta B=1} = \lambda^{ijk}_B \bar{u}_i \bar{d}_j \bar{d}_k. \]

\[ R = (-1)^{3(B-L)+2S} \]
Supersymmetric Standard Model

Superpotential (R-parity conserving)

\[ \mathcal{W}_R = y_u^{ij} \bar{u}_i Q_j \cdot H_u - y_d^{ij} \bar{d}_i Q_j \cdot H_d - y_e^{ij} \bar{e}_i L_j \cdot H_d + \mu H_u \cdot H_d, \]

Right-handed neutrino contributions to the superpotential and SUSY breaking terms

\[ y_{\nu}^{ij} \bar{\nu}_i L_j \cdot H_u, \]

\[ \lambda_{\nu}^i \bar{\nu}_i H_u \cdot H_d. \]

\[ - a_{\nu}^{ij} \bar{\nu}_i \tilde{L}_j \cdot H_u - a_{\nu R}^i \bar{\nu}_i H_u \cdot H_d, \]
The Higgs Potential

The Higgs potential then reads

\[ \mathcal{V} = (|\mu|^2 + m_{H_u}^2)(|H_u^+|^2 + |H_u^0|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) + \\
+ [b(H_u^+H_d^- - H_u^0H_d^0) + \text{h.c.}] + \frac{g^2 + g'^2}{8}(|H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \\
+ \frac{g^2}{2} |H_u^+H_d^0|^2 + H_u^0H_d^0|^2 + |\lambda_i^i\lambda_i^i| |H_u^+H_d^- - H_u^0H_d^0|^2. \] (35)

To find minima consider its part containing neutral components

\[ \mathcal{V}_n = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (bH_u^0H_d^0 + \text{h.c.}) + \\
+ \frac{g^2 + g'^2}{8} (|H_u^0|^2 - |H_d^0|^2)^2 + |\lambda_i^i\lambda_i^i| |H_u^0H_d^0|^2. \]
The Higgs Potential

Minimization conditions

\[(|\mu|^2 + m_{H_u}^2)v_u = bv_d + \frac{1}{4}(g^2 + g'^2)(v_d^2 - v_u^2)v_u - |\lambda|^2 v_d^2 v_u,\]

\[(|\mu|^2 + m_{H_d}^2)v_d = bv_u - \frac{1}{4}(g^2 + g'^2)(v_d^2 - v_u^2)v_d - |\lambda|^2 v_d v_u^2.\]

\[|\mu|^2 + m_{H_u}^2 = b \cot \beta + \frac{m_Z^2}{2} \cos 2\beta - \varepsilon^2 2m_Z^2 \cos^2 \beta\]

\[|\mu|^2 + m_{H_d}^2 = b \tan \beta - \frac{m_Z^2}{2} \cos 2\beta - \varepsilon^2 2m_Z^2 \sin^2 \beta,\]

\[m_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_u^2 + v_d^2),\]

\[\tan \beta \equiv \frac{v_u}{v_d},\]

\[\varepsilon^2 = \frac{|\lambda|^2}{g^2 + g'^2}.\]
CP-odd neutral Higgs

Mass of the CP-odd Higgs boson $A$

\[
\mathcal{V}_A = (|\mu|^2 + m_{H_u}^2)(\text{Im}H_u^0)^2 + (|\mu|^2 + m_{H_d}^2)(\text{Im}H_d^0)^2 + 2b(\text{Im}H_u^0)(\text{Im}H_d^0) +
\frac{g^2 + g'^2}{8} \left[ (\text{Re}H_u^0)^2 + (\text{Im}H_u^0)^2 - (\text{Re}H_d^0)^2 - (\text{Im}H_d^0)^2 \right]^2 +
|\lambda|^2 \left[ (\text{Re}H_u^0)^2 + (\text{Im}H_u^0)^2 \right] \left[ (\text{Re}H_d^0)^2 + (\text{Im}H_d^0)^2 \right].
\]

\[
(M_A^{sq})_{11} = |\mu|^2 + m_{H_u}^2 + \frac{g^2 + g'^2}{4}(v_u^2 - v_d^2) + \lambda^2 v_d^2 = b \cot \beta.
\]

\[
M_A^{sq} = b \begin{pmatrix} \cot \beta & 1 \\ 1 & \tan \beta \end{pmatrix}.
\]

\[
m_+^2 = 0, \quad m_-^2 = \frac{2b}{\sin 2\beta}.
\]
CP-even neutral Higggses

Masses of the CP-even Higgs bosons $h, H$

\[\mathcal{V}_H = (|\mu|^2 + m_{H_u}^2)(\text{Re}H_u^0)^2 + (|\mu|^2 + m_{H_d}^2)(\text{Re}H_d^0)^2 - 2b(\text{Re}H_u^0)(\text{Re}H_d^0) +\]
\[+ \frac{g^2 + g'^2}{8}[(\text{Re}H_u^0)^2 + (\text{Im}H_u^0)^2 - (\text{Re}H_d^0)^2 - (\text{Im}H_d^0)^2]^2 +\]
\[+ \lambda^2 \left((\text{Re}H_u^0)^2 + (\text{Im}H_u^0)^2\right) \left((\text{Re}H_d^0)^2 + (\text{Im}H_d^0)^2\right).\]

\[M_{11}^{sq} = |\mu|^2 + m_{H_u}^2 + \frac{g^2 + g'^2}{4}(2v_u^2 - v_d^2) + \lambda^2 v_d^2 = b \cot \beta + m_Z^2 \sin^2 \beta,\]

\[M_{12}^{sq} = -b - \frac{g^2 + g'^2}{2}v_u v_d + 2\lambda^2 v_u v_d = -b - \frac{1}{2}m_Z^2(1 - 4\epsilon^2) \sin 2\beta,\]

\[M_{22}^{sq} = |\mu|^2 + m_{H_d}^2 + \frac{g^2 + g'^2}{4}(2v_d^2 - v_u^2) + \lambda^2 v_u^2 = b \tan \beta + m_Z^2 \cos^2 \beta.\]
CP-even neutral Higgses

Masses of the CP-even Higgs bosons \( h, H \)

\[
M_{H,h}^{sq} = \begin{pmatrix}
 b \cot \beta + m_Z^2 \sin^2 \beta & -b - \frac{1}{2} m_Z^2 (1 - 4 \varepsilon^2) \sin 2\beta \\
-b - \frac{1}{2} m_Z^2 (1 - 4 \varepsilon^2) \sin 2\beta & b \tan \beta + m_Z^2 \cos^2 \beta
\end{pmatrix}
\]

\[
m_{H^0,h^0}^2 = \frac{1}{2} \left( m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4 m_{A^0}^2 m_Z^2 \cos^2 2\beta + \Delta_\varepsilon} \right)
\]

\[
\Delta_\varepsilon = -8 m_{A^0}^2 m_Z^2 \varepsilon^2 \sin^2 2\beta - 8 m_Z^4 \varepsilon^2 (1 - 2 \varepsilon^2) \sin^2 2\beta.
\]
CP-even neutral Higgses

Masses of the CP-even Higgs bosons $h, H$

$$m_{h^0}^2 < m_Z^2 \left( \cos^2 2\beta + 2\varepsilon^2 \sin^2 2\beta \right)$$

$$m_{h^0}^2 < m_Z^2 \cos^2 2\beta.$$ 

The situation is similar to the NMSSM case with a singlet Higgs superfield

$$m_{h^0}^2 \simeq m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta,$$
The lightest Higgs mass

The mass of the lightest Higgs boson as a function of $\tan \beta$

$$\varepsilon^2 = \frac{|\lambda|^2}{g^2 + g'^2}.$$
Charged Higgses

Masses of charged Higgs bosons

\[ \mathcal{V} = (|\mu|^2 + m_{H_u}^2)(|H_u^+|^2 + |H_u^0|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) + 
\frac{g^2 + g'^2}{8}(|H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + 
\frac{g^2}{2}|H_u^+H_d^{0\dagger} + H_u^{0\dagger}H_d^-|^2 + |\lambda_v\lambda^\dagger_v|(|H_u^+H_d^- - H_u^{0\dagger}H_d^0|^2). \] (35)

\[ M_{ch}^{sq} = \begin{bmatrix} b + v_u v_d \left( \frac{g^2}{2} - |\lambda|^2 \right) & \begin{pmatrix} \cotg \beta & 1 \\ 1 & \tan \beta \end{pmatrix} \end{bmatrix} \]

\[ m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 - 2\varepsilon^2 m_Z^2. \]
Neutrino-neutralino mixing

The R-parity breaking term

\[-\frac{1}{2} \left( (-\lambda_i^i) H_u^0 (\bar{\nu}_i \tilde{H}_d^0 + \tilde{H}_d^0 \bar{\nu}_i) + (-\lambda_i^i) H_d^0 (\bar{\nu}_i \tilde{H}_u^0 + \tilde{H}_u^0 \bar{\nu}_i) \right) \]

mixes neutrino and neutralino states and thus gives a contribution to their mass matrix.

This matrix represents quadratic (mass) terms for the following particles: four superpartners of Higgs and gauge bosons, three right-handed neutrinos (SU(2) singlets), and three left-handed neutrinos (components of SU(2)doublets)
Neutrino-neutralino mixing

Neutrino-neutralino mass matrix

\[-\frac{1}{2} \left( \tilde{G}^0 \tilde{N}^T N^T N^T \right) \begin{pmatrix}
M_{\tilde{G}^0} & M_{\tilde{G}^0 \tilde{N}} & 0 \\
M_{\tilde{G}^0 \tilde{N}}^T & M_{\tilde{N}} & M_D \\
0 & M_D^T & 0
\end{pmatrix} \begin{pmatrix}
\tilde{G}^0 \\
\tilde{N} \\
N
\end{pmatrix} + \text{h.c.},\]

\[M_{\tilde{G}^0 \tilde{N}} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
-\lambda^1 v_u & -\lambda^2 v_u & -\lambda^3 v_u \\
-\lambda^1 v_d & -\lambda^2 v_d & -\lambda^3 v_d
\end{pmatrix} = m_Z \sqrt{\frac{2}{g^2 + g'^2}} \begin{pmatrix}
0 \\
0 \\
-\lambda^i v \sin \beta \\
-\lambda^i v \cos \beta
\end{pmatrix}\]
Neutrino masses

In order to explain the huge difference between the neutrino mass scale (eVs) and the mass scale of other fermions (GeVs for quarks) one may assume the elements of $M_N$ are very large (about GUT scale). Then the mass matrix has 3 small eigenvalues ("seesaw mechanism"). In this scenario neutrino is a Majorana particle.

\[-\frac{1}{2}(\tilde{G}^0 T \bar{N}^T N T) \begin{pmatrix} M_{\tilde{G}^0} & M_{\tilde{G}^0 N} & 0 \\ M_{\tilde{G}^0 N}^T & M_N & M_D \\ 0 & M_D^T & 0 \end{pmatrix} \begin{pmatrix} \tilde{G}^0 \\ \bar{N} \\ N \end{pmatrix} + h.c.,\]
Neutrino masses

Another possibility is $M_N = 0$ and Yukawa couplings are very small. However, it is not enough to ensure the smallness of neutrino masses in general case for significant values of $\varepsilon$. Nevertheless the neutrino mass matrix still can have small eigenvalues for a certain parameter set. This requirement gives very strong constraints on the model parameters. In this scenario neutrino is a “pseudo-Dirac” particle.

$$-\frac{1}{2}(\tilde{G}^0 \bar{N}^T N^T) \begin{pmatrix} M_{\tilde{G}^0 N} & M_{\tilde{G}^0 N}^T & 0 \\ M_T^0 & M_{\tilde{G}^0 N}^T & M_D^T \\ 0 & M_D^T & 0 \end{pmatrix} \begin{pmatrix} \tilde{G}^0 \\ \bar{N} \\ N \end{pmatrix} + \text{h.c.},$$
Constraints on model parameters

Region excluded by non-observation of the light Higgs boson

\[ \varepsilon^2 = \frac{|\lambda|^2}{g^2 + g'^2} . \]
Constraints on model parameters
Conclusions

The toy R-parity broken supersymmetric model with right handed singlet neutrino superfield has been considered

It is shown that Higgs masses are modified due to new couplings

\[ m_{H^0, h^0}^2 = \frac{1}{2} \left( m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_{A^0}^2 m_Z^2 \cos^2 2\beta + \Delta_\varepsilon} \right) \]

\[ m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 - 2\varepsilon^2 m_Z^2. \]

\[ \varepsilon^2 = \frac{|\lambda|^2}{g^2 + g'^2}. \]

The famous MSSM inequality for the lightest Higgs mass is modified

\[ m_{h^0}^2 < m_Z^2 \left( \cos^2 2\beta + 2\varepsilon^2 \sin^2 2\beta \right) \]
Conclusions

The new contribution could be as large as loop corrections and leads to constraints on the parameter space of the model.

More detailed analysis is needed. The work is in progress …