Heavy quark physics
in the covariant quark model

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Hadron Structure’13
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Covariant quark confined model

Form factors for the semileptonic and rare heavy meson decays

Light baryons and their electromagnetic interactions

Rare baryon decays $\Lambda_b \to \Lambda \ell^+ \ell^-$

$X(3872)$-meson as a tetraquark state

Summary and outlook
Collaboration

- **Almaty** (M. Dineykhan, G.G. Saidullaeva)
- **Bratislava** (S. Dubnicka, A.Z. Dubnickova, A. Liptaj)
- **Dubna** (M. A. Ivanov)
- **Mainz** (J. G. Körner)
- **Napoli** (P. Santorelli)
- **Tübingen** (T. Gutsche, V. E. Lyubovitskij)
- **Valparaiso** (S. G. Kovalenko)
Covariant quark model of hadrons

- Main assumption: hadrons interact via quark exchange only

- Interaction Lagrangian

\[ \mathcal{L}_{\text{int}} = g_H \cdot H(x) \cdot J_H(x) \]
Covariant quark model of hadrons

- Main assumption: hadrons interact via quark exchange only

- Interaction Lagrangian

\[ \mathcal{L}_{\text{int}} = g_H \cdot H(x) \cdot J_H(x) \]

- Quark currents

\[
J_M(x) = \int dx_1 \int dx_2 \, F_M(x; x_1, x_2) \cdot \bar{q}_{f_1}^a(x_1) \Gamma_M q_{f_2}^a(x_2) \quad \text{Meson}
\]

\[
J_B(x) = \int dx_1 \int dx_2 \int dx_3 \, F_B(x; x_1, x_2, x_3) \times \Gamma_1 q_{f_1}^{a_1}(x_1) \left[ \varepsilon^{a_1a_2a_3} q_{f_2}^{T \, a_2}(x_2) C \Gamma_2 q_{f_3}^{a_3}(x_3) \right] \quad \text{Baryon}
\]

\[
J_T(x) = \int dx_1 \ldots \int dx_4 \, F_T(x; x_1, \ldots, x_4) \times \left[ \varepsilon^{a_1a_2c} q_{f_1}^{T \, a_1}(x_1) C \Gamma_1 q_{f_2}^{a_2}(x_2) \right] \cdot \left[ \varepsilon^{a_3a_4c} \bar{q}_{f_3}^{T \, a_3}(x_3) \Gamma_2 C \bar{q}_{f_4}^{a_4}(x_4) \right] \quad \text{Tetraquark}
\]
Compositeness condition $Z_H = 0$

- A composite field and its constituents are introduced as elementary particles.
- The transition of a composite field to its constituents is provided by the interaction Lagrangian.
- The renormalization constant $Z^{1/2}$ is the matrix element between a physical state and the corresponding bare state. If there is a stable bound state which we wish to represent by introducing a quasi-particle $H$, then elementary particle must have renormalization factor $Z$ equal to zero.

\[
Z_H^{1/2} = <H_{\text{bare}}|H_{\text{dressed}}> = 0
\]

We use the compositeness condition to determine the hadron-quark coupling constant, e.g. in the case of mesons

\[
Z_M = 1 - \tilde{\Pi}'(m^2_M) = 0
\]

where $\tilde{\Pi}(p^2)$ is the meson mass operator.
The vertex functions and quark propagators

- Translational invariance for the vertex function

\[ F_H(x + a, x_1 + a, x_2 + a) = F_H(x, x_1, x_2), \quad \forall a. \]

- Our choice:

\[ F_B(x, x_1, \ldots, x_n) = \delta^{(4)}(x - \sum_{i=1}^{n} w_i x_i) \Phi_H\left(\sum_{i<j}(x_i - x_j)^2\right) \]

where \( w_i = m_i / \sum_{i} m_i \).

- The quark propagators

\[ S_q(x_1 - x_2) = \int \frac{d^4 k}{(2\pi)^4 i} \frac{e^{-i k(x_1 - x_2)}}{m_q - k} \]
The matrix elements

- The matrix elements are described by a set of the Feynman diagrams which are convolution of the quark propagators and vertex functions.

- Let $\Pi$ be the matrix element corresponding to the Feynman diagram:

  $j$ external momenta;
  $n$ quark propagators;
  $\ell$ loop integrations;
  $m$ vertices.

  In the momentum space it will be represented as

  \[
  \Pi(p_1, \ldots, p_j) = \int [d^4k]^{\ell} \prod_{i_1=1}^{m} \Phi_{i_1+n} \left( -K_{i_1+n}^2 \right) \prod_{i_3=1}^{n} S_{i_3} (\tilde{k}_{i_3} + \tilde{p}_{i_3})
  \]

  \[
  K_{i_1+n}^2 = \sum_{i_2} \left( \tilde{k}_{i_1+n}^{(i_2)} + \tilde{p}_{i_1+n}^{(i_2)} \right)^2
  \]

  $\tilde{k}_i$ are linear combinations of the loop momenta $k_i$
  $\tilde{p}_i$ are linear combinations of the external momenta $p_i$
Infrared confinement

- Use the Schwinger representation of the propagator:

\[
\frac{m + \not{k}}{m^2 - k^2} = (m + \not{k}) \int_0^\infty d\alpha \exp[-\alpha(m^2 - k^2)]
\]

- Choose a simple Gaussian form for the vertex function

\[
\Phi(-K^2) = \exp\left(\frac{K^2}{\Lambda^2}\right)
\]

where the parameter \( \Lambda \) characterizes the hadron size.

- We imply that the loop integration \( k \) proceed over Euclidean space:

\[
k^0 \rightarrow e^{i\frac{\pi}{2}} k_4 = ik_4, \quad k^2 = (k^0)^2 - \vec{k}^2 \rightarrow -k_E^2 \leq 0.
\]

- We also put all external momenta \( p \) to Euclidean space:

\[
p^0 \rightarrow e^{i\frac{\pi}{2}} p_4 = ip_4, \quad p^2 = (p^0)^2 - \vec{p}^2 \rightarrow -p_E^2 \leq 0
\]

so that the quadratic momentum form in the exponent becomes negative-definite and the loop integrals are absolutely convergent.
Convert the loop momenta in the numerator into derivatives over external momenta:

\[ k_i^{\mu} e^{2kr} = \frac{1}{2} \frac{\partial}{\partial r_i \mu} e^{2kr}, \]

Move the derivatives outside of the loop integrals.

Calculate the scalar loop integral:

\[
\prod_{i=1}^{n} \int \frac{d^4k_i}{i\pi^2} e^{kAk + 2kr} = \prod_{i=1}^{n} \int \frac{d^4k_i^E}{\pi^2} e^{-k_E A k_E - 2k_E r_E} = \frac{1}{|A|^2} e^{-r A^{-1} r}
\]

where a symmetric \( n \times n \) real matrix \( A \) is positive-definite.

Use the identity

\[
P \left( \frac{1}{2} \frac{\partial}{\partial r} \right) e^{-r A^{-1} r} = e^{-r A^{-1} r} P \left( \frac{1}{2} \frac{\partial}{\partial r} - A^{-1} r \right)
\]

to move the exponent to the left.
Infrared confinement

- Employ the commutator

\[
\left[ \frac{\partial}{\partial r_i}, r_j \right] = \delta_{ij} g_{\mu\nu}
\]

to make differentiation in

\[
P \left( \frac{1}{2} \frac{\partial}{\partial r} - A^{-1} r \right)
\]

for any polynomial \( P \). The necessary commutations of the differential operators are done by a FORM program.

- One obtains

\[
\Pi = \int_0^\infty d^n \alpha \ F(\alpha_1, \ldots, \alpha_n),
\]

where \( F \) stands for the whole structure of a given diagram.
Infrared confinement

The set of Schwinger parameters $\alpha_i$ can be turned into a simplex by introducing an additional $t$–integration via the identity

$$1 = \int_0^\infty dt \, \delta(t - \sum_{i=1}^n \alpha_i)$$

leading to

$$\Pi = \int_0^\infty dt \int_0^1 d^n \alpha \, \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, \ldots, t\alpha_n).$$
Infrared confinement

- Cut off the upper integration at $1/\lambda^2$

$$\Pi^c = \frac{1}{\lambda^2} \int_0^1 dt \int_0^{t^{n-1}} d^n \alpha \delta \left( 1 - \sum_{i=1}^n \alpha_i \right) F(t\alpha_1, \ldots, t\alpha_n)$$

- The infrared cut-off has removed all possible thresholds in the quark loop diagram.

- We take the cut-off parameter $\lambda$ to be the same in all physical processes.

Infrared confinement

- An example of a scalar one–loop two–point function:

\[ \Pi_2(p^2) = \int \frac{d^4k_E}{\pi^2} \frac{e^{-s k_E^2}}{[m^2 + (k_E + \frac{1}{2} p_E)^2][m^2 + (k_E - \frac{1}{2} p_E)^2]} \]

where the numerator factor \( e^{-s k_E^2} \) comes from the product of nonlocal vertex form factors of Gaussian form. \( k_E, p_E \) are Euclidean momenta \( (p_E^2 = - p^2) \).

- Doing the loop integration one obtains

\[ \Pi_2(p^2) = \int_0^\infty dt \frac{t}{(s + t)^2} \int_0^1 d\alpha \ exp \left\{ - t [m^2 - \alpha(1 - \alpha)p^2] + \frac{st}{s + t} \left( \alpha - \frac{1}{2} \right)^2 p^2 \right\} \]

A branch point at \( p^2 = 4m^2 \).
Infrared confinement

- By introducing a cut-off in the $t$–integration one obtains

$$\Pi_c^2(p^2) = \frac{1}{\lambda^2} \int_0^1 dt \frac{t}{(s + t)^2} \int_0^1 d\alpha \exp \left\{ -t [m^2 - \alpha(1 - \alpha)p^2] + \frac{st}{s + t} \left( \alpha - \frac{1}{2} \right)^2 p^2 \right\}$$

where the one–loop two–point function $\Pi_c^2(p^2)$ no longer has a branch point at $p^2 = 4m^2$.

- The confinement scenario also allows to include all possible both two-quark and multi-quark resonance states in our calculations.
In order to guarantee local invariance of the strong interaction Lagrangian one multiplies each quark field $q(x_i)$ in nonlocal quark current $J_H(x)$ with a gauge field exponential:

$$q_i(x_i) \rightarrow e^{-i e q_1 I(x_i, x, P)} q_i(x_i) \quad \text{where} \quad I(x_i, x, P) = \int_x^{x_i} dz_\mu A^\mu(z).$$

The path $P$ connects the end-points of the path integral. We use the path-independent definition of the derivative of $I(x, y, P)$:

$$\lim_{dx_\mu \rightarrow 0} dx_\mu \frac{\partial}{\partial x_\mu} I(x, y, P) = \lim_{dx_\mu \rightarrow 0} [I(x + dx, y, P') - I(x, y, P)]$$

where the path $P'$ is obtained from $P$ by shifting the end-point $x$ by $dx$. The definition leads to the key rule

$$\frac{\partial}{\partial x_\mu} I(x, y, P) = A_\mu(x)$$

which in turn states that the derivative of the path integral $I(x, y, P)$ does not depend on the path $P$ originally used in the definition.
Subtleties: gauging

Diagrams describing $V \rightarrow \gamma$ transition:

\[ M^\mu_\nu(p) = \int \frac{d^4k}{4\pi^2i} \Phi_V(-k^2) \text{tr}\left( \gamma^\mu S(k + \frac{1}{2}p)\gamma^\nu S(k - \frac{1}{2}p) \right) \]

\[ M^\mu_\nu(p) = -\int \frac{d^4k}{4\pi^2i} \left(2k + \frac{1}{2}p\right)^\mu \int_0^1 d\alpha \Phi'_V \left( -\alpha(k + \frac{1}{2}p)^2 - (1 - \alpha)k^2 \right) \times \text{tr}\left( \gamma^\nu S(k) \right) \]
If $p = 0$ then the second diagram maybe transfered to the first one by using integration by parts

$$
\int \frac{d^4 k}{4\pi^2 i} \frac{\partial}{\partial k^{\mu}} \left\{ \Phi_V \left( - k^2 \right) \text{tr} \left( \gamma^\nu S(k) \right) \right\} =
$$

$$= \int \frac{d^4 k}{4\pi^2 i} \left\{ - 2k^\mu \Phi'_V \left( - k^2 \right) \text{tr} \left( \gamma^\nu S(k) \right) \right. + \left. \Phi_V \left( - k^2 \right) \text{tr} \left( \gamma^\mu S(k) \gamma^\nu S(k) \right) \right\} = 0.
$$
Input values for the leptonic decay constants $f_H$ (in MeV) and our least-squares fit values.

<table>
<thead>
<tr>
<th>Fit Values</th>
<th>PDG/LAT</th>
<th>This work</th>
<th>PDG/LAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_\pi$</td>
<td>128.7</td>
<td>130.4 ± 0.2</td>
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</tr>
<tr>
<td>$f_K$</td>
<td>156.1</td>
<td>156.1 ± 0.8</td>
<td></td>
</tr>
<tr>
<td>$f_D$</td>
<td>205.9</td>
<td>206.7 ± 8.9</td>
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<tr>
<td>$f_{D_s}$</td>
<td>257.5</td>
<td>257.5 ± 6.1</td>
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<tr>
<td>$f_B$</td>
<td>191.1</td>
<td>192.8 ± 9.9</td>
<td></td>
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<tr>
<td>$f_{B_s}$</td>
<td>234.9</td>
<td>238.8 ± 9.5</td>
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<tr>
<td>$f_{B_c}$</td>
<td>489.0</td>
<td>489 ± 5</td>
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<tr>
<td>$f_\rho$</td>
<td>221.1</td>
<td>221 ± 1</td>
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</table>
Model parameters

Input values for some basic electromagnetic decay widths and our least-squares fit values (in keV).

<table>
<thead>
<tr>
<th>Process</th>
<th>Fit Values</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0 \to \gamma\gamma$</td>
<td>$5.06 \times 10^{-3}$</td>
<td>$(7.7 \pm 0.4) \times 10^{-3}$</td>
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<tr>
<td>$\eta_c \to \gamma\gamma$</td>
<td>$1.61$</td>
<td>$1.8 \pm 0.8$</td>
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<tr>
<td>$\rho^\pm \to \pi^\pm\gamma$</td>
<td>$76.0$</td>
<td>$67 \pm 7$</td>
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<tr>
<td>$\omega \to \pi^0\gamma$</td>
<td>$672$</td>
<td>$703 \pm 25$</td>
</tr>
<tr>
<td>$K^*\pm \to K^\pm\gamma$</td>
<td>$55.1$</td>
<td>$50 \pm 5$</td>
</tr>
<tr>
<td>$K^*0 \to K^0\gamma$</td>
<td>$116$</td>
<td>$116 \pm 10$</td>
</tr>
<tr>
<td>$D^*\pm \to D^\pm\gamma$</td>
<td>$1.22$</td>
<td>$1.5 \pm 0.5$</td>
</tr>
<tr>
<td>$J/\psi \to \eta_c\gamma$</td>
<td>$1.43$</td>
<td>$1.58 \pm 0.37$</td>
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</table>
Model parameters

The results of the fit for the values of quark masses $m_{q_i}$, the infrared cutoff parameter $\lambda$ and the size parameters $\Lambda_{H_i}$ (all in GeV).

<table>
<thead>
<tr>
<th></th>
<th>$m_u$</th>
<th>$m_s$</th>
<th>$m_c$</th>
<th>$m_b$</th>
<th>$\lambda$</th>
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<tr>
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<td>0.235</td>
<td>0.424</td>
<td>2.16</td>
<td>5.09</td>
<td>0.181</td>
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<td>GeV</td>
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<th>$\Lambda_\pi$</th>
<th>$\Lambda_K$</th>
<th>$\Lambda_D$</th>
<th>$\Lambda_{D_s}$</th>
<th>$\Lambda_B$</th>
<th>$\Lambda_{B_s}$</th>
<th>$\Lambda_{B_c}$</th>
<th>$\Lambda_\rho$</th>
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<tr>
<td></td>
<td>0.87</td>
<td>1.04</td>
<td>1.47</td>
<td>1.57</td>
<td>1.88</td>
<td>1.95</td>
<td>2.42</td>
<td>0.61</td>
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<table>
<thead>
<tr>
<th></th>
<th>$\Lambda_\omega$</th>
<th>$\Lambda_\phi$</th>
<th>$\Lambda_{J/\psi}$</th>
<th>$\Lambda_{K^*}$</th>
<th>$\Lambda_{D^*}$</th>
<th>$\Lambda_{D_s^*}$</th>
<th>$\Lambda_{B^*}$</th>
<th>$\Lambda_{B_s^*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.47</td>
<td>0.88</td>
<td>1.48</td>
<td>0.72</td>
<td>1.16</td>
<td>1.17</td>
<td>1.72</td>
<td>1.71</td>
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</table>
The study of heavy flavor physics: motivation

- To determine the Cabibbo-Kobayashi-Maskawa matrix elements.
- To provide insights into the origin of flavor and CP-violation.
- To look for new physics beyond the standard model.
- The subject to study are heavy hadrons containing a $b$– or a $c$–quark and their weak decays.
The study of heavy flavor physics: **motivation**

- To determine the Cabibbo-Kobayashi-Maskawa matrix elements.
- To provide insights into the origin of flavor and CP-violation.
- To look for new physics beyond the standard model.
- The subject to study are heavy hadrons containing a $b$– or a $c$–quark and their weak decays.

- The main idea in the theoretical studies of heavy-flavor decays is to separate short-distance (perturbative) QCD dynamics from long-distance (nonperturbative) hadronic effects.

- One uses the so-called *naive* factorization approach which is based on the weak effective Hamiltonian describing quark and lepton transitions in terms of local operators that are multiplied by Wilson coefficients.

- The Wilson coefficients characterize the short-distance dynamics and may be reliably evaluated by perturbative methods.
The study of heavy flavor physics: motivation

- The calculation of the hadronic matrix elements of local operators between initial and final states require nonperturbative methods. One needs to know how hadrons are constructed from quarks.

- Technically, any matrix element of a local operator may be expressed in terms of a set of scalar functions which are referred to as form factors.

- A variety of theoretical approaches have been used to evaluate the hadronic form factors:
The study of heavy flavor physics: motivation

- The calculation of the hadronic matrix elements of local operators between initial and final states require nonperturbative methods. One needs to know how hadrons are constructed from quarks.

- Technically, any matrix element of a local operator may be expressed in terms of a set of scalar functions which are referred to as form factors.

- A variety of theoretical approaches have been used to evaluate the hadronic form factors:
  - The light-cone sum rule (LCSR) approach (Braun, Ball, Khodjamirian et al.)
  - Dyson-Schwinger equations in QCD (C.D. Roberts et al.)
  - A relativistic quark model (Faustov, Galkin et al.)
  - The constituent quark model with dispersion relations (Melikhov et al.)
  - A QCD relativistic potential model (Ladisa, et al.)
  - A QCD sum rule analysis (P. Colangelo et al.)
Semileptonic $B \rightarrow D$ transition

$O^\mu = \gamma^\mu - \gamma^\mu \gamma^5$

$\phi_B \left( - (k + w_u p_1)^2 \right) \quad \phi_{D(D^*)} \left( - (k + w'_u p_2)^2 \right)$

$w_u = \frac{m_u}{m_u + m_b}$

$w'_u = \frac{m_u}{m_u + m_c}$
Semileptonic $B \rightarrow D$ transition

\[ O^\mu = \gamma^\mu - \gamma^\mu \gamma^5 \]

\[ \phi_B \left( - (k + w_u p_1)^2 \right) \quad \phi_{D(D^*)} \left( - (k + w'_u p_2)^2 \right) \]

\[ w_u = \frac{m_u}{m_u + m_b} \quad w'_u = \frac{m_u}{m_u + m_c} \]

Heavy quark limit: $m_H = m_Q + E$, $m_Q \rightarrow \infty$; $\Lambda_B = \Lambda_D = \Lambda_{D^*}$.
Isgur-Wise function

\[
\frac{1}{m_i - k - p_i} \rightarrow - \frac{1 + \gamma_i}{2} \cdot \frac{1}{k v_i + E}, \quad \nu = \frac{p}{m}
\]
Isgur-Wise function

\[ \frac{1}{m_i - k - p_i'} \rightarrow - \frac{1 + y_i}{2} \cdot \frac{1}{kv_i + E}, \quad v = \frac{p}{m} \]

\[ M_{BD}^\mu(p_1, p_2) = f_+(q^2)(p_1 + p_2)^\mu + f_-(q^2)(p_1 - p_2)^\mu, \]

\[ f_\pm = \frac{M_c \pm M_b}{2\sqrt{M_b M_c}} \cdot \xi(w), \]

The compositeness condition \( Z_H = 0 \) provides the correct normalization

\[ \xi(w = 1) = 1 \]
Form factors for semileptonic, nonleptonic and rare $B (B_s)$ meson decays


\[ \phi_{B(B_s)} \left( - (k + r_1)^2 \right) \quad \phi_{P(V)} \left( - (k + r_2)^2 \right) \]

\[ r_i = \frac{m_{q_3}}{m_{q_i} + m_{q_3}} p_i \]
The definition of the form factors

\[
\langle P'_{[\bar{q}_3 q_2]}(p_2) | \bar{q}_2 O^\mu q_1 | P_{[\bar{q}_3 q_1]}(p_1) \rangle = F_+(q^2) P^\mu + F_-(q^2) q^\mu
\]

\[
\langle P'_{[\bar{q}_3 q_2]}(p_2) | \bar{q}_2 (\sigma^{\mu\nu} q_\nu) q_1 | P_{[\bar{q}_3 q_1]}(p_1) \rangle = \frac{i}{m_1 + m_2} \left( q^2 P^\mu - q \cdot P q^\mu \right) F_T(q^2)
\]

\[
\langle V(p_2, \epsilon_2)_{[\bar{q}_3 q_2]} | \bar{q}_2 O^\mu q_1 | P_{[\bar{q}_3 q_1]}(p_1) \rangle =
\]

\[
= \frac{\epsilon^\dagger_\nu}{m_1 + m_2} \left( -g^{\mu\nu} P \cdot q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) \\
+ i \epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta V(q^2) \right)
\]

\[
\langle V(p_2, \epsilon_2)_{[\bar{q}_3 q_2]} | \bar{q}_2 (\sigma^{\mu\nu} q_\nu (1 + \gamma^5)) q_1 | P_{[\bar{q}_3 q_1]}(p_1) \rangle =
\]

\[
= \epsilon^\dagger_\nu \left( - (g^{\mu\nu} - q^\mu q^\nu / q^2) P \cdot q a_0(q^2) + (P^\mu P^\nu - q^\mu P^\nu P \cdot q / q^2) a_+(q^2) \\
+ i \epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta g(q^2) \right)
\]

\[
P = p_1 + p_2 , \quad q = p_1 - p_2 , \quad \epsilon_2^\dagger \cdot p_2 = 0 .
\]
Form factors

$B-\pi$: $F_+(q^2)$

$B-K$: $F_+(q^2)$

$B-\pi$: $F_T(q^2)$

$B-K$: $F_T(q^2)$
Form factors

$B_s\Phi: A_1(q^2)$

$B_s\Phi: A_2(q^2)$

$B_s\Phi: V(q^2)$

$B_s\Phi: T_1(q^2)$
Form factors

\[ F_{\text{VDM}}^{B\pi}(q^2) = \frac{F_+^{B\pi}(0)}{m_{B^*}^2 - q^2}. \]
Nonleptonic $B_s$ decays

- The modes $B_s \to D_s^- D_s^+$, $D_s^*^- D_s^+ + D_s^- D_s^{*+}$, $D_s^*^- D_s^{*+}$ give the largest contribution to $\Delta\Gamma_s \equiv \Gamma_L - \Gamma_H$ for the $B_s - \bar{B}_s$ system.

- The mode $B_s \to J/\psi \phi$ is color–suppressed but it is interesting for the search of possible CP-violating new physics effects in $B_s - \bar{B}_s$ mixing.

- Nonleptonic $B_s^0 \to J/\psi \eta (\eta')$ decays were observed by Belle Coll.:
  

- Their decay widths were calculated in our approach by
  
The effective Hamiltonian

Current-current diagrams

tree OCD one-loop

OCD penguin
The effective Hamiltonian

- The effective Hamiltonian describing the $B_s$ nonleptonic decays:

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^\dagger \sum_{i=1}^{6} C_i Q_i$$

- Current-current diagrams:

$$Q_1 = (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{s}_{a_2} c_{a_1})_{V-A} \quad Q_2 = (\bar{c}_{a_1} b_{a_1})_{V-A}, (\bar{s}_{a_2} c_{a_2})_{V-A}$$

- QCD penguin diagram:

$$Q_3 = (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V-A} \quad Q_4 = (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V-A}$$

$$Q_5 = (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V+A} \quad Q_6 = (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V+A}$$

$$\langle \bar{q}q \rangle_{V-A} = \bar{q}\gamma^\mu (1 - \gamma^5) q \quad \text{left–chiral current}$$

$$\langle \bar{q}q \rangle_{V+A} = \bar{q}\gamma^\mu (1 + \gamma^5) q \quad \text{right–chiral current}$$
Nonleptonic $B_s$ decays

Annihilation diagram
Nonleptonic $B_s$ decays
Calculated branching ratios (%) of the $B_s$ nonleptonic decays.

<table>
<thead>
<tr>
<th>Process</th>
<th>This work</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s \to D_s^- D_s^+$</td>
<td>1.65</td>
<td>$1.04^{+0.29}_{-0.26}$</td>
</tr>
<tr>
<td>$B_s \to D_s^- D_s^{<em>+} + D_s^{</em>-} D_s^+$</td>
<td>2.40</td>
<td>$2.8 \pm 1.0$</td>
</tr>
<tr>
<td>$B_s \to D_s^{<em>-} D_s^{</em>+}$</td>
<td>3.18</td>
<td>$3.1 \pm 1.4$</td>
</tr>
<tr>
<td>$B_s \to J/\psi \phi$</td>
<td>0.16</td>
<td>$0.14 \pm 0.05$</td>
</tr>
</tbody>
</table>
Nucleon as three-quark state: Lagrangian


Lagrangian describing the interaction of proton (antiproton) with its constituents:

$$\mathcal{L}_{\text{int}}^P(x) = g_N \bar{p}(x) \cdot J_p(x) + \text{h.c.}$$

The interpolating three-quark current:

$$J_p(x) = \int dx_1 \int dx_2 \int dx_3 F_N(x; x_1, x_2, x_3) J_{3q}^{(p)}(x_1, x_2, x_3)$$

$$J_{3q}^{(p)}(x_1, x_2, x_3) = \Gamma^A \gamma^5 a_1(x_1) \cdot [\epsilon^{a_1a_2a_3} u^{a_2}(x_2) C \Gamma_A u^{a_3}(x_3)].$$

There are two kinds of three-quark currents:

$$\Gamma^A \otimes \Gamma_A = \gamma^\alpha \otimes \gamma_\alpha \quad \text{(vector)} \quad \Gamma^A \otimes \Gamma_A = \frac{1}{2} \sigma^\alpha\beta \otimes \sigma_\alpha\beta \quad \text{(tensor)}$$

We consider a general linear superposition:

$$J_N = xJ_N^T + (1 - x)J_N^V, \quad N = p, n$$

with a mixing parameter $x \ (0 \leq x \leq 1)$.
Electromagnetic vertex function of proton

(a)

(b)

(c)

(d)
Static properties of nucleons

Parameters:

- a superposition of the V– and T–currents of nucleons with $x = 0.8$
- the size parameter of the nucleon we take $\Lambda_N = 0.5$ GeV.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Our results</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_p$ (in n.m.)</td>
<td>2.96</td>
<td>2.793</td>
</tr>
<tr>
<td>$\mu_n$ (in n.m.)</td>
<td>-1.83</td>
<td>-1.913</td>
</tr>
<tr>
<td>$r_E^p$ (fm)</td>
<td>0.805</td>
<td>0.8768 ± 0.0069</td>
</tr>
<tr>
<td>$&lt; r_E^2 &gt;^n$ (fm$^2$)</td>
<td>-0.121</td>
<td>-0.1161 ± 0.0022</td>
</tr>
<tr>
<td>$r_M^p$ (fm)</td>
<td>0.688</td>
<td>0.777 ± 0.013 ± 0.010</td>
</tr>
<tr>
<td>$r_M^n$ (fm)</td>
<td>0.685</td>
<td>$0.862^{+0.009}_{-0.008}$</td>
</tr>
</tbody>
</table>
Electromagnetic form factors

$G_M^p / \mu_p$ (mixing)

$G_M^n / \mu_n$ (mixing)

$G_E^p$ (mixing)

$(4m_N^2 / q^2) (G_E^n / \mu_n)$, $q^2 = -Q^2$, (mixing)
Rare baryon decays $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$: motivation

- The decay $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ is complement to the well–analyzed rare meson decays $B \rightarrow K(\ast) \ell^+ \ell^-$ etc. to study the short– and long–distance dynamics of rare decays induced by the transition $\mathbf{b} \rightarrow \mathbf{s} \ell^+ \ell^-$. 

- The experimental measurements:

  \[
  \mathcal{B}(\Lambda_b \rightarrow \Lambda \mu^+ \mu^-) = (1.73 \pm 0.42\text{(stat)} \pm 0.55\text{(syst)}) \cdot 10^{-6}
  \]


  \[
  \mathcal{B}(\Lambda_b \rightarrow \Lambda \mu^+ \mu^-) = (0.96 \pm 0.16\text{(stat)} \pm 0.13\text{(syst)} \pm 0.21\text{(norm)}) \cdot 10^{-6}
  \]

  RAaij et al. [LHCb Collaboration], arXiv:1306.2577 [hep-ex].

- A number of theoretical papers use the SM (penguin) operators and their non–Standard Model extensions to describe the short distance dynamics.

- Nonperturbative approaches to calculate the transition matrix element $\langle \Lambda | O_i | \Lambda_b \rangle$. 
Lagrangian and 3-quark currents


\[ \mathcal{L}_{\text{int}}^\Lambda(x) = g_\Lambda \bar{\Lambda}(x) \cdot J_\Lambda(x) + g_\Lambda \bar{J}_\Lambda(x) \cdot \Lambda(x) \]

\[ J_\Lambda(x) = \int dx_1 \int dx_2 \int dx_3 \ F_\Lambda(x; x_1, x_2, x_3) \ J_{3q}^{(\Lambda)}(x_1, x_2, x_3) \]

\[ J_{3q}^{(\Lambda)}(x_1, x_2, x_3) = Q^{a_1}(x_1) \cdot \epsilon^{a_1a_2a_3} \ u^T \ a_2(x_2) \ C \gamma^5 \ d^{a_3}(x_3) \]

\[ Q = s, c, b \]

The vertex function is chosen in the form

\[ F_\Lambda(x; x_1, x_2, x_3) = \delta^{(4)}(x - \sum_{i=1}^{3} w_i x_i) \ \Phi_\Lambda \left( \sum_{i<j} (x_i - x_j)^2 \right) \]

\[ w_i = \frac{m_i}{m_1 + m_2 + m_3} \]
The rare baryon decays $\Lambda_b \rightarrow \Lambda + \ell^+\ell^-$ and $\Lambda_b \rightarrow \Lambda + \gamma$

- The effective Hamiltonian leads to the quark decay amplitudes $b \rightarrow s\ell^+\ell^-$:

$$M(b \rightarrow s\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha \lambda_t}{2\pi} \left\{ C_{\text{eff}}^9 (\bar{s}O^\mu b) (\bar{\ell}\gamma_\mu \ell) + C_{10} (\bar{s}O^\mu b) (\bar{\ell}\gamma_\mu \gamma_5 \ell) \right\} - \frac{2}{q^2} C_{\text{eff}}^7 \left[ m_b \left( \bar{s} i\sigma^{\mu q} (1 + \gamma^5) b \right) + O(m_s) \right] (\bar{\ell}\gamma_\mu \ell).$$

- and $b \rightarrow s\gamma$:

$$M(b \rightarrow s\gamma) = -\frac{G_F}{\sqrt{2}} \frac{e \lambda_t}{4\pi^2} C_{\text{eff}}^7 \left[ m_b \left( \bar{s} i\sigma^{\mu q} (1 + \gamma^5) b \right) + O(m_s) \right] \epsilon_\mu,$$

where $\lambda_t \equiv V_{ts}^\dagger V_{tb}$.

- The Wilson coefficient $C_{\text{eff}}^9$ effectively takes into account, first, the contributions from the four-quark operators $Q_i (i = 1, \cdots, 6)$ and, second, the nonperturbative effects (long–distance contributions) coming from the $c\bar{c}$-resonance contributions what are, as usual, parametrized by a Breit-Wigner ansatz.
The rare baryon decays $\Lambda_b \to \Lambda + \ell^+\ell^-$ and $\Lambda_b \to \Lambda + \gamma$

The hadronic matrix elements are expanded in terms of dimensionless form factors:

\[
\begin{align*}
\langle B_2 | \bar{s} \gamma^\mu b | B_1 \rangle &= \bar{u}_2(p_2) \left[ f^V_1(q^2) \gamma^\mu - f^V_2(q^2) i\sigma^{\mu q_1} + f^V_3(q^2) q_1^\mu \right] u_1(p_1) \\
\langle B_2 | \bar{s} \gamma^\mu \gamma^5 b | B_1 \rangle &= \bar{u}_2(p_2) \left[ f^A_1(q^2) \gamma^\mu - f^A_2(q^2) i\sigma^{\mu q_1} + f^A_3(q^2) q_1^\mu \right] \gamma^5 u_1(p_1) \\
\langle B_2 | \bar{s} i\sigma^{\mu q} b | B_1 \rangle &= \bar{u}_2(p_2) \left[ f^{TV}_1(q^2)(\gamma^\mu q_1 - q_1^\mu q_1) - f^{TV}_2(q^2) i\sigma^{\mu q_1} \right] u_1(p_1) \\
\langle B_2 | \bar{s} i\sigma^{\mu q_1} \gamma^5 b | B_1 \rangle &= \bar{u}_2(p_2) \left[ f^{TA}_1(q^2)(\gamma^\mu q_1 - q_1^\mu q_1) - f^{TA}_2(q^2) i\sigma^{\mu q_1} \right] \gamma^5 u_1(p_1)
\end{align*}
\]

where $q = p_1 - p_2$ and $q_1 = q/M_1$. 
The fit of the size parameters

- We use the same values of the quark masses and the infrared cut-off as in meson sector.

- We determine the set of size parameters $\Lambda_{\Lambda_s}$, $\Lambda_{\Lambda_c}$ and $\Lambda_{\Lambda_b}$ by fitting data on the magnetic moment of the $\Lambda$-hyperon and the branching ratios of the semileptonic decays $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ and $\Lambda_b \rightarrow \Lambda_c \ell^- \bar{\nu}_\ell$ by a one-parameter fit to these values.

- With the choice of dimensional parameters in GeV

  $$\Lambda_{\Lambda_s} = 0.490 \quad \Lambda_{\Lambda_c} = 0.864 \quad \Lambda_{\Lambda_b} = 0.569$$

  we get:

  $$\mu_{\Lambda_s} = -0.73 \quad \mu_{\Lambda_s}^{\text{expt}} = -0.613 \pm 0.004$$

  $$\mu_{\Lambda_c} = +0.39$$

  $$\mu_{\Lambda_b} = -0.06$$
The fit of the size parameters

Branching ratios of semileptonic decays of heavy baryons in \%.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Our results</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_c \to \Lambda e^+ \nu_e$</td>
<td>2.0</td>
<td>2.1 ± 0.6</td>
</tr>
<tr>
<td>$\Lambda_c \to \Lambda \mu^+ \nu_\mu$</td>
<td>2.0</td>
<td>2.0 ± 0.7</td>
</tr>
<tr>
<td>$\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$</td>
<td>6.6</td>
<td>6.5$^{+3.2}_{-2.5}$</td>
</tr>
<tr>
<td>$\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu$</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\tau$</td>
<td>1.8</td>
<td></td>
</tr>
</tbody>
</table>

Asymmetry parameter $\alpha$ in the semileptonic decays of heavy baryons.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Our results</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_c \to \Lambda e^+ \nu_e$</td>
<td>0.828</td>
<td>0.86 ± 0.04</td>
</tr>
<tr>
<td>$\Lambda_c \to \Lambda \mu^+ \nu_\mu$</td>
<td>0.825</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$</td>
<td>0.831</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu$</td>
<td>0.831</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\tau$</td>
<td>0.731</td>
<td></td>
</tr>
</tbody>
</table>
The rare baryon decays $\Lambda_b \to \Lambda + \ell^+ \ell^-$ and $\Lambda_b \to \Lambda + \gamma$

Our results:

$$\mathcal{B}(\Lambda_b \to \Lambda \mu^+ \mu^-) = 1.0 \cdot 10^{-6}$$


to be compared with the recent LHCb data:

$$\mathcal{B}(\Lambda_b \to \Lambda \mu^+ \mu^-) = (0.96 \pm 0.16({\text{stat}}) \pm 0.13({\text{syst}}) \pm 0.21({\text{norm}})) \cdot 10^{-6}$$

RAaij et al. [LHCb Collaboration], arXiv:1306.2577 [hep-ex].

$$\mathcal{B}(\Lambda_b \to \Lambda \gamma) = 0.4 \cdot 10^{-5} \quad \text{(experimental upper bound < } 130 \cdot 10^{-5})$$
The angular decay distribution for the cascade decay $\Lambda_b \rightarrow \Lambda(\rightarrow p\pi^-)\gamma$

$$\frac{d\Gamma(\Lambda_b \rightarrow \Lambda(\rightarrow p\pi^-)\gamma)}{d \cos \theta_B} = \text{Br}(\Lambda \rightarrow p\pi^-) \frac{1}{2} \Gamma(\Lambda_b \rightarrow \Lambda\gamma)(1 + \alpha_B \tilde{P}^\Lambda_z \cos \theta_B)$$

where $\alpha_B$ is the asymmetry parameter in the decay $\Lambda \rightarrow p + \pi^-$ for which we take the experimental value $\alpha_B = 0.642 \pm 0.013$.

$$\Gamma(\Lambda_b \rightarrow \Lambda\gamma) = \frac{\alpha}{2} \left( \frac{G_F m_b |\lambda_t| C_7^{\text{eff}}}{4\pi^2 \sqrt{2}} \right)^2 \frac{(M_1^2 - M_2^2)^3}{M_1^3} \left[ \left( f_{2TV}(0) \right)^2 + \left( f_{2TA}(0) \right)^2 \right]$$

The $z$–component of the polarization of the $\Lambda$ is given by

$$\tilde{P}_z^\Lambda = -2 \frac{f_{2TV}(0)f_{2TA}(0)}{(f_{2TV}(0))^2 + (f_{2TA}(0))^2}$$

One can show that $f_{2TV}(0) \equiv f_{2TA}(0)$. Therefore, $\tilde{P}_z^\Lambda \equiv -1$ and finally

$$\frac{1}{\Gamma_{\text{tot}}} \frac{d\Gamma(\Lambda_b \rightarrow \Lambda(\rightarrow p\pi^-)\gamma)}{d \cos \theta_B} = \text{Br}(\Lambda \rightarrow p\pi^-) \frac{1}{2} \text{Br}(\Lambda_b \rightarrow \Lambda\gamma)(1 - \alpha_B \cos \theta_B)$$
The angular decay distribution for the cascade decay $\Lambda_b \to \Lambda(p\pi^-)\ell^+\ell^-$.

**Figure:** Definition of angles $\theta$, $\theta_B$ and $\chi$ in the cascade decay $\Lambda_b \to \Lambda(p\pi^-) + J_{\text{eff}}(\ell^+\ell^-)$. 

\[ \text{Definition of angles $\theta$, $\theta_B$ and $\chi$ in the cascade decay} \]

\[ \Lambda_b \to \Lambda(p\pi^-) + J_{\text{eff}}(\ell^+\ell^-). \]
The differential rate of the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$

$$
\frac{d\Gamma(\Lambda_b \to \Lambda\ell^+\ell^-)}{dq^2} = \frac{v^2}{2} \cdot \left( U^{11+22} + L^{11+22} \right) + \frac{2m_\ell^2}{q^2} \cdot \frac{3}{2} \cdot \left( U^{11} + L^{11} + S^{22} \right)
$$

The total rate is obtained by $q^2$–integration in the range

$$
4m_\ell^2 \leq q^2 \leq (M_1 - M_2)^2
$$

The short notations:

$$
X^{mm'} = \frac{1}{2} \frac{G_F^2}{(2\pi)^3} \left( \frac{\alpha|\lambda_t|}{2\pi} \right)^2 \frac{p_2^2}{12} \frac{q^2 v}{M_1^2} H_X^{mm'},
$$

where $\lambda_t = V_{ts}^+ V_{tb} = 0.041$ and $v = \sqrt{1 - 4m_\ell^2/q^2}$ is the lepton velocity in the $(\ell^+\ell^-)$ CM frame.
Lepton–side decay distribution for the cascade decay $\Lambda_b \rightarrow \Lambda(\rightarrow p\pi^-)\ell^+\ell^-$

$$\frac{d\Gamma(\Lambda_b \rightarrow \Lambda \ell^+ \ell^-)}{dq^2 d\cos \theta} = v^2 \cdot \left[ \frac{3}{8} (1 + \cos^2 \theta) \cdot \frac{1}{2} U^{11+22} + \frac{3}{4} \sin^2 \theta \cdot \frac{1}{2} L^{11+22} \right]$$

$$- v \cdot \frac{3}{4} \cos \theta \cdot P^{12} + \frac{2m^2_{\ell}}{q^2} \cdot \frac{3}{4} \cdot \left[ U^{11} + L^{11} + S^{22} \right]$$

One can define a lepton–side forward–backward asymmetry $A_{FB}^\ell$ by $A_{FB}^\ell = (F - B)/(F + B)$ where $F$ and $B$ denote the rates in the forward and backward hemispheres.

$$A_{FB}^\ell(q^2) = -\frac{3}{2} \frac{v \cdot P^{12}}{v^2 \cdot (U^{11+22} + L^{11+22}) + \frac{2m^2_{\ell}}{q^2} \cdot 3 \cdot (U^{11} + L^{11} + S^{22})}.$$

The integrated forward–backward asymmetry is defined as the ratio of the integrals of the numerator and denominator over $q^2$ in the full kinematical region.
\[ \Lambda \text{-polarization and hadron–side decay distribution for the cascade decay } \Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^- \]

\[ \frac{d\Gamma(\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-)}{dq^2 \, d \cos \theta_B} = Br(\Lambda \to p\pi^-) \cdot \frac{1}{2} \frac{d\Gamma(\Lambda_b \to \Lambda \ell^+\ell^-)}{dq^2} \times \left( 1 + \alpha_B P^\Lambda_z \cos \theta_B \right) \]

The \(z\)-component of the polarization of the daughter baryon \(\Lambda\):

\[ P^\Lambda_z = \frac{v^2 \cdot \left( P^{11+22}_p + L^{11+22}_p \right) + \frac{2m^2}{q^2} \cdot 3 \cdot \left( P^{11} + L^{11}_p + S^{22}_p \right)}{v^2 \cdot \left( U^{11+22} + L^{11+22} \right) + \frac{2m^2}{q^2} \cdot 3 \cdot \left( U^{11} + L^{11} + S^{22} \right)} \]

The forward–backward asymmetry is simply related to the polarization \(P^\Lambda_z\) via

\[ A_{FB}^h(q^2) = \frac{\alpha_B}{2} \cdot P^\Lambda_z(q^2) \]
Asymmetries $A_{FB}^l$ and $A_{FB}^h$ with (without) long–distance contributions

<table>
<thead>
<tr>
<th>Mode</th>
<th>$A_{FB}^l$</th>
<th>$A_{FB}^h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_b \rightarrow \Lambda e^+ e^-$</td>
<td>$3.2 \times 10^{-10}$</td>
<td>$(1.2 \times 10^{-8})$</td>
</tr>
<tr>
<td>$\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$</td>
<td>$1.7 \times 10^{-4}$</td>
<td>$(8.0 \times 10^{-4})$</td>
</tr>
<tr>
<td>$\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$</td>
<td>$5.9 \times 10^{-4}$</td>
<td>$(9.6 \times 10^{-4})$</td>
</tr>
</tbody>
</table>
A narrow charmonium-like state $X(3872)$ was observed in the exclusive decay process:

$$B^+ \rightarrow K^+ \pi^+ \pi^- J/\psi$$


- $X$-mass is close to $D^0 - D^{*0}$ mass threshold:

$$M_X = 3871.68 \pm 0.17 \text{ MeV} , \quad \text{PDG'12}$$

$$M_{D^0} + M_{D^{*0}} = 3871.81 \pm 0.25 \text{ MeV}$$

- Its width $\Gamma_X \leq 1.2 \text{ MeV}$ at 90% CL.
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- Its width \( \Gamma_X \leq 1.2 \text{ MeV} \) at 90% CL.

- The state was confirmed in B-decays by BaBar experiment


and in \( p\bar{p} \) production by Tevatron experiments CDF and DØ.

D. E. Acosta et al. [CDF Collaboration] Phys. Rev. Lett. 93, 072001 (2004);

X(3872)-meson: short introduction

- From the observation of decays $X(3872) \rightarrow J/\psi \gamma$ reported by both Belle and BaBar collaborations and from the angular analysis performed by CDF experiment it was shown that the only quantum numbers $J^{PC} = 1^{++}$ or $2^{-+}$ are compatible with data.

  K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0505037; hep-ex/0505038
  B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 74, 071101 (2006)

- The observation of decays into $D^0\bar{D}^0\pi^0$ by Belle and BaBar collaborations allows one to exclude the choice $2^{-+}$ because the near-threshold decay $X \rightarrow D^0\bar{D}^0\pi^0$ is expected to be strongly suppressed for $J = 2$.


- The quantum numbers of the $X(3872)$ meson were determined from the analysis of angular correlations in $B^+ \rightarrow X(3872)K^+$ decays, where $X(3872) \rightarrow \pi^+\pi^-J/\psi$ and $J/\psi \rightarrow \mu^+\mu^-.$


The quantum numbers of the $X(3872)$ are

$J^{PC} = 1^{++}$
X(3872)-meson: short introduction

- Belle collaboration has reported evidence for the decay mode $X \rightarrow \pi^+ \pi^- \pi^0 J/\psi$ dominated by the sub-threshold decay $X \rightarrow \omega J/\psi$.

  K. Abe et al., [Belle Collaboration], arXiv:hep-ex/0505037, hep-ex/0505038

- It was found that the branching ratio of this mode is almost the same as of $X \rightarrow \pi^+ \pi^- J/\psi$ decay:

$$\frac{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 1.0 \pm 0.4 \text{ (stat)} \pm 0.3 \text{ (syst).}$$

- It implies strong isospin violation because the three-pion decay proceeds via intermediate $\omega$-meson with isospin 0 whereas the two-pion decay proceeds via intermediate $\rho$-meson with isospin 1.
X(3872)-meson: short introduction

- The two-pion decay via intermediate $\rho$-meson is very difficult to explain by using an interpretation of the X(3872) as simple $c\bar{c}$ charmonium state with isospin 0.

- The possible candidate from $\bar{c}c$-spectroscopy:

$$\chi_{c1}(2^3P_1) - \text{state with } J^{PC} = 1^{++}$$

BUT the value of its mass varies from 3925 up to 3953 MeV. Also the decay width calculated in various models is too large.

- The X(3872) IS NOT the pure $\bar{c}c$-state
The two-pion decay via intermediate $\rho$-meson is very difficult to explain by using an interpretation of the $X(3872)$ as simple $c\bar{c}$ charmonium state with isospin 0.

The possible candidate from $\bar{c}c$-spectroscopy:

$$\chi_{c_1}(2^3P_1) - \text{state with } J^{PC} = 1^{++}$$

BUT the value of its mass varies from 3925 up to 3953 MeV. Also the decay width calculated in various models is too large.

The $X(3872)$ IS NOT the pure $\bar{c}c$-state

- a molecule bound state $D^0\bar{D}^*0$ with small binding energy
- a tetraquark state composed from a diquark and antidiquark
- threshold cusps
- hybrids and glueballs
An interpretation of the $X(3872)$ as a tetraquark was suggested in


$$X_q \rightarrow [cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}, \quad (q = u, d)$$

Isospin breaking: the state $X_u$ breaks isospin symmetry maximally:

$$X_u = \frac{1}{\sqrt{2}} \left\{ \frac{X_u + X_d}{\sqrt{2}} + \frac{X_u - X_d}{\sqrt{2}} \right\}.$$
The physical states are the mixing of $X_u$ and $X_d$

\[
X_l \equiv X_{\text{low}} = X_u \cos \theta + X_d \sin \theta, \\
X_h \equiv X_{\text{high}} = -X_u \sin \theta + X_d \cos \theta.
\]

The mixing angle $\theta$ is supposed to be found from the known ratio of the two-pion (via $\rho$) and three-pion (via $\omega$) decay widths.
The physical states are the mixing of $X_u$ and $X_d$

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\]

The mixing angle $\theta$ is supposed to be found from the known ratio of the two-pion (via $\rho$) and three-pion (via $\omega$) decay widths.

We have performed independent analysis of the X(3872)-meson considered as a tetraquark state in the framework of the covariant quark model with infrared confinement.
X(3872)-meson as a tetraquark state: Lagrangian


▷ An effective interaction Lagrangian

\[ \mathcal{L}_{\text{int}} = g_X X_q \mu (x) \cdot J_{X_q}^{\mu} (x), \quad (q = u, d). \]

▷ The nonlocal version of the four-quark interpolating current

\[
J_{X_q}^{\mu} (x) = \int dx_1 \ldots \int dx_4 \delta (x - \sum_{i=1}^{4} w_i x_i) \Phi_x \left( \sum_{i<j} (x_i - x_j)^2 \right) J_{4q}^{\mu} (x_1, \ldots, x_4)
\]

\[
J_{4q}^{\mu} = \frac{1}{\sqrt{2}} \epsilon_{abc} \left[ q_a (x_4) C \gamma^5 c_b (x_1) \right] \epsilon_{dec} \left[ \bar{q}_d (x_3) \gamma^{\mu} C \bar{c}_e (x_2) \right] + (\gamma^5 \leftrightarrow \gamma^{\mu}),
\]

\[
w_1 = w_2 = \frac{m_c}{2(m_q + m_c)} \equiv \frac{w_c}{2}, \quad w_3 = w_4 = \frac{m_q}{2(m_q + m_c)} \equiv \frac{w_q}{2}.
\]
The coupling constant $g_X$ is determined from the compositeness condition

$$Z_X = 1 - \Pi_X'(M_X^2) = 0$$

where $\Pi_X(p^2)$ is the scalar part of the vector-meson mass operator.
Since the $X(3872)$ lies nearly the respective thresholds in both cases,

$$m_X - (m_{J/\psi} + m_\rho) = -0.90 \pm 0.41 \text{ MeV},$$
$$m_X - (m_{D^0} + m_{D^{*0}}) = -0.30 \pm 0.34 \text{ MeV}$$

the intermediate $\rho(\omega)$ and $D^*$ mesons should be taken off-shell.
The narrow width approximation

\[
\frac{d\Gamma(X \to J/\psi + n\pi)}{dq^2} = \frac{1}{8 m_X^2 \pi} \cdot \frac{1}{3} |M(X \to J/\psi + \nu^0)|^2 \\
\times \frac{\Gamma_{\nu^0} m_{\nu^0}}{\pi} \frac{p^*(q^2)}{(m_{\nu^0}^2 - q^2)^2 + \Gamma_{\nu^0}^2 m_{\nu^0}^2} \text{Br}(\nu^0 \to n\pi),
\]

\[
\frac{d\Gamma(X_u \to \bar{D}^0 D^0 \pi^0)}{dq^2} = \frac{1}{2 m_X^2 \pi} \cdot \frac{1}{3} |M(X_u \to \bar{D}^0 D^{*0})|^2 \\
\times \frac{\Gamma_{D^{*0}} m_{D^{*0}}}{\pi} \frac{p^*(q^2) \mathcal{B}(D^{*0} \to D^0 \pi^0)}{(m_{D^{*0}}^2 - q^2)^2 + \Gamma_{D^{*0}}^2 m_{D^{*0}}^2},
\]
Strong decay widths

- Two new adjustable parameters: $\theta$ and $\Lambda_X$.

- The ratio
  \[
  \frac{\Gamma(X_u \to J/\psi + 3\pi)}{\Gamma(X_u \to J/\psi + 2\pi)} \approx 0.25
  \]
  is very stable under variation of $\Lambda_X$.

- Using this result and the central value of the experimental data
  \[
  \frac{\Gamma(X_{l,h} \to J/\psi + 3\pi)}{\Gamma(X_{l,h} \to J/\psi + 2\pi)} \approx 0.25 \cdot \left(\frac{1 \pm \tan \theta}{1 \mp \tan \theta}\right)^2 \approx 1
  \]
  gives $\theta \approx \pm 18.4^\circ$ for $X_l$ (" + ") and $X_h$ (" − "), respectively.

- This is in agreement with the results obtained by both Maiani: $\theta \approx \pm 20^\circ$ and Nielsen: $\theta \approx \pm 23.5^\circ$. 
Strong decay widths

\[
\frac{\Gamma(X \rightarrow D^0 \bar{D}^0 \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 
4.5 \pm 0.2 & \text{theor} \\
10.5 \pm 4.7 & \text{expt} 
\end{cases}
\]
Radiative $X$-decay

S. Dubnica, A. Z. Dubnickova, M. A. Ivanov, J. G. Koerner, P. Santorelli and G. G. Saidullaeva,
Phys. Rev. D 84, 014006 (2011)
Radiative $X$-decay

The on-mass shell conditions

$$\varepsilon_{X}^{\mu} p_{\mu} = 0, \quad \varepsilon_{J/\psi}^{\nu} q_{1\nu} = 0, \quad \varepsilon_{\gamma}^{\rho} q_{2\rho} = 0$$

leave us five Lorentz structures:

$$T_{\mu\rho\nu}(q_1, q_2) = \varepsilon_{q_2\mu\nu\rho}(q_1 \cdot q_2) W_1 + \varepsilon_{q_1 q_2\nu\rho} q_{1\mu} W_2 + \varepsilon_{q_1 q_2\mu\rho} q_{2\nu} W_3$$

$$+ \varepsilon_{q_1 q_2\mu\nu} q_{1\rho} W_4 + \varepsilon_{q_1 \mu\nu\rho}(q_1 \cdot q_2) W_5.$$

Using the gauge invariance condition

$$q_{2\rho} T_{\mu\rho\nu} = (q_1 \cdot q_2) \varepsilon_{q_1 q_2\mu\nu} (W_4 + W_5) = 0$$

one has $W_4 = -W_5$ which reduces the set of independent covariants to four. However, there are two nontrivial relations among the four covariants which can be derived by noting that the tensor

$$T_{\mu[\nu_1\nu_2\nu_3\nu_4\nu_5]} = g_{\mu\nu_1} \varepsilon_{\nu_2\nu_3\nu_4\nu_5} + \text{cycl.}(\nu_1\nu_2\nu_3\nu_4\nu_5)$$

vanishes in four dimensions since it is totally antisymmetric in the five indices $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5)$. 
Radiative $X$-decay

The two conditions reduce the set of independent covariants to two. This is the appropriate number of independent covariants since the photon transition is described by two independent amplitudes as e.g. by the $E1$ and $M2$ transition amplitudes. One has

$$\Gamma(X \rightarrow \gamma J/\psi) = \frac{1}{12\pi} \frac{|\vec{q}_2|^2}{m_X^2} \left(|H_L|^2 + |H_T|^2\right) = \frac{1}{12\pi} \frac{|\vec{q}_2|^2}{m_X^2} \left(|A_{E1}|^2 + |A_{M2}|^2\right),$$

where the helicity amplitudes $H_L$ and $H_T$ are expressed in terms of the Lorentz amplitudes as

$$H_L = i \frac{m_X^2}{m_{J/\psi}} |\vec{q}_2|^2 \left[W_1 + W_3 - \frac{m_{J/\psi}}{m_X|\vec{q}_2|} W_4\right],$$

$$H_T = -im_X |\vec{q}_2|^2 \left[W_1 + W_2 - \left(1 + \frac{m_{J/\psi}}{m_X|\vec{q}_2|}\right) W_4\right],$$

$$|\vec{q}_2| = \frac{m_X^2 - m_{J/\psi}^2}{2m_X}.$$

The $E1$ and $M2$ multipole amplitudes are obtained via

$$A_{E1/M2} = (H_L \mp H_T)/\sqrt{2}.$$
If one takes $\Lambda_X \in (3, 4)$ GeV with the central value $\Lambda_X = 3.5$ GeV then our prediction for the ratio of widths reads

$$\left. \frac{\Gamma(X \rightarrow J/\psi + \gamma)}{\Gamma(X \rightarrow J/\psi + 2\pi)} \right|_{\text{theor}} = 0.15 \pm 0.03$$

which fits very well the experimental data from the Belle Collaboration

$$\frac{\Gamma(X \rightarrow \gamma + J/\psi)}{\Gamma(X \rightarrow J/\psi 2\pi)} = \begin{cases} 
0.14 \pm 0.05 & \text{Belle} \\
0.22 \pm 0.06 & \text{BaBar}
\end{cases}$$
Summary and outlook

- We have presented a refined covariant quark model which includes infrared confinement of quarks.

- We have calculated the transition form factors of the heavy $B$ and $B_s$ mesons to light pseudoscalar and vector mesons, which are needed as ingredients for the calculation of the semileptonic, nonleptonic, and rare decays of the $B$ and $B_s$ mesons. Our form factor results hold in the full kinematical range of momentum transfer.

- We have made use of the calculated form factors to calculate the nonleptonic decays $B_s \rightarrow D_s \bar{D}_s$, ... and $B_s \rightarrow J/\psi \phi$, which have been widely discussed recently in the context of $B_s - \bar{B}_s$-mixing and CP violation.

- We have applied our approach to baryon physics by using the same values of the constituent quark masses and infrared cutoff as in meson sector.

- We have calculated the nucleon magnetic moments and charge radii and also electromagnetic form factors at low energies.
Summary and outlook

- We have explored the rare baryon decays $\Lambda_b \rightarrow \Lambda + \ell^+ \ell^-$ and $\Lambda_b \rightarrow \Lambda + \gamma$.
- We have studied the properties of the $X(3872)$ as a tetraquark.
- We have calculated the strong decays $X \rightarrow J/\psi + \rho(\rightarrow 2\pi)$, $X \rightarrow J/\psi + \omega(\rightarrow 3\pi)$, $X \rightarrow D + \bar{D}^*(\rightarrow D\pi)$ and electromagnetic decay $X \rightarrow \gamma + J/\psi$.
- The comparison with available experimental data allows one to conclude that the $X(3872)$ can be a tetraquark state.