SIGNIFICANCE OF NON-PERTURBATIVE INPUT TO TMD GLUON DENSITY IN HARD PROCESSES AT LHC

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OUTLINE

1. Inclusive spectra of charge hadrons in p-p within soft QCD model including gluon

2. Gluon distribution in proton

3. Modified un-integrated gluon distribution

4. CCFM-evolution and structure functions

5. Summary

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SOFT PP -> h X

The inclusive spectrum is presented in the following form:

\[ \rho (x=0, p_t) = \rho_q (x=0, p_t) + \rho_g (x=0, p_t) \]

Here

\[ \rho_q = g \left( \frac{s}{s_o} \right)^\Delta \varphi_q ; \varphi_q (0, p_t) = A_q \exp (-b_q p_t) \]

\[ \rho_g = g \left[ \left( \frac{s}{s_o} \right)^\Delta - \sigma_{nd} \right] \varphi_g ; \varphi_g (0, p_t) = \sqrt{p_t} A_g \exp (-b_g p_t) \]

\[ A_q = 11.91 \pm 0.39, \quad b_q = 7.29 \pm 0.11 \]
\[ A_g = 3.76 \pm 0.13, \quad b_g = 3.51 \pm 0.02 \]
\[ g \approx 21 \text{ mb}, \quad \Delta \approx 0.12 \]


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In the light cone dynamics the proton has a general decomposition:

\[ |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \ldots \]


One-Pomeron exchange (left) and the cut one-Pomeron exchange (right); P-proton, g-gluon, h-hadron produced in PP.
THE CUT ONE-POMERON EXCHANGE

\[ \rho(x, p_{ht}) = ((F(x_+, p_{ht}) F(x_-, p_{ht}))^2 \]

Here

\[ F(x_+, p_{ht}) = \int dx_1 \int d^2k_{1t} f_{Rq}(x_1, k_{1t}) G^h_q \left( \frac{x_+}{x_1}, p_{ht} - k_{1t} \right) \]

where

\[ G^h_q(z, k_t) = z D^h_q(z, k_t) \quad f_q = g \otimes P_{g-qq} \]

where \( P_{g-qq} \) is the splitting function of a gluon to the quark-antiquark pair

$s^{1/2} = 7 \text{ TeV}$

- soft QCD (quarks)
- soft QCD (gluons)
- SQCD (quarks + gluons)
- Exp. data (CMS, ATLAS)
UN-INTEGRATED GLUON DISTRIBUTION IN PROTON

\[ xA \left( x, k_t^2, Q_0^2 \right) = \frac{3\sigma_0}{4\pi^2} R_0^2(x) k_t^2 \exp \left( - R_0^2(x) k_t^2 \right), \]

where \( R_0 = C_1 (x/x_0)^{\lambda/2} \), \( C_1 = 1/\text{GeV} \)


MODIFIED UGD AT $Q_0$

$$xg(x, k_t, Q_0) = C_0 C_3 (1 - x)^b_s \left( R_0^2(x) k_t^2 + C_2 (R_0(x) k_t)^q \right)$$

$$\exp \left( - R_0(x) k_t - d (R_0(x) k_t)^3 \right) ,$$

where

$$C_0 = \frac{3 \sigma_0}{\left( 4\pi^2 \alpha_s (Q_0^2) \right)}$$

The coefficient $C_3$ is found from the relation

$$xg(x, Q_0^2) = \frac{Q_0^2}{0} xg(x, k_t^2, Q_0^2) dk_t^2$$


At $k_t \to 0$ our UGD goes to zero as $k_t^a$ where $a < 1$


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CCFM evolution equation

\[ f_g(x, k_T^2, q^2) = f_g^0(x, k_T^2, Q_0^2) \Delta_s(q^2, Q_0^2) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \times \]

\[ \theta(q - zq) \Delta_s(q^2, q^2) P_{gqq}(z, q^2, k_T^2) f_g \left( \frac{x}{z}, k_T^2, q^2 \right) \]

Here \( k'_T = q(1-z)/z + k_T \) and the Sudakov form factor \( \Delta_s(q_1^2, q_2^2) \) describes the probability of no radiation between \( q_2 \) and \( q_1 \), \( P_{gqq} \) is the splitting function, \( f_g \) is the gluon density. The first term means the contribution of non resolvable branchings between the starting scale \( Q_0 \) and the factorization scale \( q \).

MODIFICATION OF U.G.D. AT LARGE $k_T$

We construct the new U.G.D. matching their form at low $k_T$ ($k_T < 2-3$ GeV/c) to the one, which is the exact solution of the BFKL outside of the saturation region.

$$x_{g_1}(x, k_T, Q_0) = x_{g_0}(x, k_T, Q_0) + F_M(x, k_T, Q_0)P_1(x, k_T)$$

Here $x_{g_1}$ is the new U.G.D., $x_{g_0}$ is our old U.G.D., $P_1$ is the BFKL solution at $k_T > 1$ GeV/c, $F_M$ is the matching function of $x_{g_0}$ to $P_1$. 

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Solution of the linear BFKL equation

\[ P_1(k_T, Y) = C_{-1} \frac{\Lambda}{k_T} \frac{\exp[(\alpha_p - 1)Y]}{\sqrt{14\alpha_s N_c \zeta(3)Y}} \exp\left( -\frac{\pi}{14\alpha_s N_c \zeta(3)Y} \ln^2 \frac{k_T}{\Lambda} \right) \]

\( \alpha_p \) is the intercept of the subcritical Pomeron, \( Y = \ln(1/x) \)

For the initial conditions, as the two gluon exchange approximation \( C_{-1} \sim \alpha_s^2 \). This solution is valid at \( k_T < \Lambda(1/x)^{\alpha_s N_c} \)

Our matching function

\[ F_M(x, k_T, Q_0) = B \left( \frac{x}{x_0} \right)^d \exp\left( -a R_0 / k_T \right) \]

where \( R_0 = \left( \frac{x}{x_0} \right)^\lambda \), \( B, d, a \) are parameters, which were found from matching of our old U.G.D. to the Kovchegov’s solution
Spectrum of $\pi^-$ - mesons produced in pp collision

Gluon density as a function of $k_T^2$

Left: our input $ugd$ at $Q_0 \sim 1\text{-}2\text{ GeV}$, the solid line is the new $ugd$, the dashed line is our old $ugd$.

Right: the $ugd$ at $\mu^2 = 100\text{ GeV}^2$, solid line is our new $ugd$, and the dash-dotted line corresponds to the set $A_0$ (H. Jung).

$f_g^{(0)}(x, k_T^2, Q_0^2) \rightarrow f_g^{(0)}(x, k_T^2, Q_0^2) + f_g^{(k)}(x, k_T^2)$
Solid line corresponds to $\mu_R = Q$, the dash top line is for $\mu_R = Q/2$, the bottom line corresponds to $\mu_R = 2Q$. Circles are the ZEUS data, squares are H1 data.
$F_2^b (x,Q^2)$

Solid line corresponds to $\mu_R = Q$, the dash top line is for $\mu_R = Q/2$, the bottom line corresponds to $\mu_R = 2Q$. Circles are the H1 data, squares are H1 data. Dotted line is the calculations using the set A0, Hannes Jung hep-ph/0411287
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Within the $k_T$-factorization approach we have

$$\sigma = \int \frac{|\hat{M}|^2}{16\pi (x_1 x_2 s)^2} f_g(x_1, k_{1T}^2, \mu^2) f_g(x_2, k_{2T}^2, \mu^2) dp_{1T}^2 dk_{1T}^2 dk_{2T}^2 dy_1 dy_2 \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi}$$
B$^+$-meson production in p-p collision at $s^{1/2} = 8$ TeV

Solid histogram is our new calculation, the dashed one is results obtained using set A0, see Hannes Jung, Proc. 12th Int. Workshop DIS’2004, Strbske Pleso, Slovakia, 2004, hep-ph/0411287.
PP-\(\rightarrow\)b-jet +X at \(s^{1/2} = 8\) TeV
Muon production from semi-leptonic decay of b-jets

Solid histogram is our new calculation, the dashed one is results obtained using set A0, see Hannes Jung, Proc. 12th Int. Workshop DIS’2004, Strbske Pleso, Slovakia, 2004, hep-ph/0411287.
**D* - meson production in pp collision at $s^{1/2} = 8$ TeV**

Solid histogram is our new calculation, the dashed one is results obtained using set A0, see Hannes Jung, Proc. 12th Int. Workshop DIS’2004, Strbske Pleso, Slovakia, 2004, hep-ph/0411287.
Higgs-boson production in pp collision

\[ \mathcal{L}_{ggH} = \frac{\alpha_s}{12\pi} \left( G_F \sqrt{2} \right)^{1/2} G_{\mu\nu}^{a} G^{a\mu\nu} H. \]

\[ T_{ggH}^{\mu\nu,ab}(k_1, k_2) = i\delta^{ab} \frac{\alpha_s}{3\pi} \left( G_F \sqrt{2} \right)^{1/2} \left[ k_2^\mu k_1^\nu - (k_1 \cdot k_2) g^{\mu\nu} \right] \]

\[ \mathcal{L}_{H\gamma\gamma} = \frac{\alpha}{8\pi} A \left( G_F \sqrt{2} \right)^{1/2} F_{\mu\nu} F^{\mu\nu} H \]

\[ A = A_W(\tau_W) + N_c \sum \frac{Q_j^2}{Q_f} A_f(\tau_f) \]

\[ \tau_f = \frac{m_H^2}{4m_f^2}, \quad \tau_W = \frac{m_H^2}{4m_W^2} \]

Higgs-boson production in pp at $s^{1/2} = 8$ TeV

Solid histogram is our calculation, the dashed one is results using set A0, see Hannes Jung, Proc. 12th Int. Workshop DIS’2004Strbske Pleso, Slovakia, 2004. hep-ph/0411287. Solid histogram is results of Hannes Jung, hep-ph/0411287; the light lilac area corresponds to NNLO+NNLL.

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SUMMARY

1. The new TMD gluon density is proposed at initial $Q_0 = 1.1$ GeV/c.
   and their parameters are verified by the description of the LHC data on the hadron spectra in the soft kinematical region.

2. The CCFM evolution equation was solved using the proposed TMD u.g.d. at starting $Q_0^2$.

3. The CCFM-evolved u.g.d. results in a satisfactory description of the H1 and ZEUS data on $F_L, F_{2b}$.

4. The modification of the u.g.d. at large $k_T$ is suggested matching the solution of the BFKL equation at $k_T > 1$ GeV/c and our u.g.d., which was obtained at $k_T < 1$ GeV/c.

5. The CCFM-evolved new u.g.d. results in a satisfactory description of hard production of heavy flavour jets and Higgs bosons.

6. The application of the new u.g.d. to the analysis of these processes allows us to describe rather well the azimuthal correlations of two b-jets.

7. The connection between the soft processes at LHC and small x-physics at HERA has been confirmed using the new input for the gluon density.

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THANK YOU VERY MUCH FOR YOUR ATTENTION!
New input for TMD gluon density

\[ f_g^{(0)}(x, k_T^2, Q_0^2) \rightarrow f_g^{(0)}(x, k_T^2, Q_0^2) + f_g^{(k)}(x, k_T^2) \]
Longitudinal structure function

\[ f_{g}^{(0)}(x, k_T^2, Q_0^2) \rightarrow f_{g}^{(0)}(x, k_T^2, Q_0^2) + f_{g}^{(k)}(x, k_T^2) \]
Longitudinal structure function as a function of $x$

The solid lines correspond to the proposed CCFM – evolved TMD gluon density; the dashed curves mean the contribution from the our non evolved gluon density; the dottet lines correspond to the CCFM-evolved GBW g.d.

Kt-factorization

Photo-production cross section

\[ \sigma = \int \frac{dz}{z} d^2 k_t \sigma_{part} \left( \frac{x}{z}, k_t^2 \right) F \left( z, k_t^2 \right) \]

Here \( F \left( z, k_t^2 \right) \) is the un-integrated parton density function, \( \sigma_{part} \left( x/z, k_t^2 \right) \) is the partonic cross section.

Classification scheme:

\( xF \left( x, k_t^2 \right) \) is used by BFKL

\( xA \left( x, k_t^2, \bar{Q}^2 \right) \) describes the CCFM type UGD with an additional factorization scale \( \bar{Q} \) (such as \( \alpha_s \left( \bar{Q}^2 \right) \leq 1 \))

\( xG \left( x, k_t^2 \right) \) describes the DGLAP type UGD

DESY, March 26, 2015
Longitudinal structure function within the $kt$-factorization

$$F_L(x,Q^2) = \int_0^1 \frac{dz}{z} \int_0^{Q^2} dk_t^2 \sum_{i=u,d,s} e_i^2 C^g_L \left( \frac{x}{z}, Q^2, m_i^2, k_t^2 \right) \phi_g(z, k_t^2),$$

$$\phi_g(x, k_t^2) = x g(x, k_t^2), \quad x g(x, Q^2) = x g(x, Q_0^2) + \int_{Q_0^2}^{Q^2} dk_t^2 \phi_g(x, k_t^2)$$


QCD@LHC2014
$F_L$ as a function of $Q^2$ at $W=276$ GeV and $\mu_R^2 = 127 Q^2$

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Effective dipole cross section

\[
\sigma_{\text{dipole}}(x, r) = \sigma_0 \left\{ 1 - \exp \left( - \frac{r^2}{4R^2_0(x)} \right) \right\}
\]

\[
\sigma_{\text{GBW}}(x, r) = \sigma_0 \left\{ 1 - \exp \left( - \frac{a_1 r}{R_0(x)} - \frac{a_2 r^2}{R_0^2(x)} \right) \right\}
\]

Green line:

Red line:

DESY, March 26, 2015
Effective dipole cross section

Red line corresponds to

\[ \sigma_{dipole} = \sigma_0 \ln \left( 1 + \frac{r^2}{4R_0^2} \right) \]
The $x$-dependence of $F_L$ at $Q^2 = 2.2 \,(\text{GeV}/c)^2$ assuming

$$\mu_R^2 = KQ^2 \quad \text{and} \quad \mu_R^2 = Q^2,$$

where $K = 127$. 
Inclusive hadron production in central region and the AGK cancellation

According to the AGK, the n-Pomeron contributions to the inclusive hadron spectrum at $y=0$ are cancelled and only the one-Pomeron contributes. This was proved asymptotically, i.e., at very high energies.

Using this AGK we estimate the inclusive spectrum of the charged hadrons produced in p-p at $y=0$ as a function of the transverse momentum including the quark and gluon components in the proton.

\[
\rho_q(x=0, p_t) = \phi_q(0, p_t) \sum_{n=1}^{\infty} n\sigma_n(s) = g s^\Delta \phi_q(0, p_t)
\]

\[
\rho_g(x=0, p_t) = \phi_g(0, p_t) \sum_{n=2}^{\infty} (n-1)\sigma_n(s) =
\]

\[
\varphi_g(0, p_t)(g s^\Delta - \sigma_{nd})
\]

2012
Blue line corresponds to

\[
\sigma^{AM}_{\text{dipole}} = \sigma_0 \left\{ 1 - \exp \left[ -\frac{r^2}{4R_0^2} \ln \left( \frac{1}{\Lambda r} + e \right) \right] \right\}; \quad \Lambda = 0.24\text{GeV} = 1.2\text{fm}^{-1}; \quad R_0 = 1\text{GeV}^{-1} = 0.2\text{fm}
\]

*DESY, March 26, 2015*
Inclusive hadron production in central region and the AGK cancellation

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\[
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\]

\[
\rho_g(x = 0, p_t) = \phi_g(0, p_t) \sum_{n=1}^{\infty} (n-1)\sigma_n(s) = \\
\phi_g(0, p_t)(g s^\Lambda - \sigma_{nd})
\]

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Effective dipole cross section

Blue line corresponds to

\[ \sigma_{dipole} = \sigma_0 \frac{r^2}{4R_0^2} \ln \left(1 + \frac{4R_0^2}{r^2} \right) \]

Saturation dynamics

\[ \sigma_{dipole}^{GBW}(x, r) = \sigma_0 \left\{ 1 - \exp \left( -\frac{r^2}{4R_0^2} \right) \right\} \]

\[ R_0 = GeV^{-1}(x/x_0)^{\lambda/2} \]

Saturation becomes when \( r \sim 2R_0 \). It leads to \( \sigma_T \sim \sigma_0 \)

when \( QR_0 < 1 \) or \( Q < 1/R_0 \)

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Effective dipole cross section and unintegrated gluon distribution

\[ \sigma_{\text{dipole}}(x, r) = \frac{4\pi}{3} \int \frac{dk_t^2}{k_t^2} [1 - J_0(k_t, r)] \alpha_s x g(x, k_t) \]

Here \( \alpha_s \) is the QCD running constant, \( J_0 \) is the Bessel function of the zero order.
Structure of an event

- Multiple parton-parton interactions

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