Renormalization group for the Standard Model without dimensional regularization

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Evolution of the scalar mass: $\phi^4$

**Scalar field anomalous dimension:**  
$$\gamma_\Phi = \frac{g^2}{12(16\pi^2)^2}$$

- **Naive** (quadratic divergence in the self-energy, Susskind, 1979):  
  $$m^2(Q^2)/Q^2 = m_{ph}^2/Q^2 + \gamma_\Phi$$
- **Smart** (MS-scheme, Macfarlane & Collins 1974):  
  $$m^2(Q^2)/Q^2 = (m_{ph}^2/Q^2)^{1-\gamma_\Phi}$$
- **Correct** (Gell-Mann—Law scheme, GP, 2010):  
  $$\frac{m^2(Q^2)}{Q^2} = \frac{\gamma_\Phi}{1-4\gamma_\Phi \log \frac{Q^2}{m_{ph}^2}}$$

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in the Standard Model, the Landau pole in the scalar mass squared is replaced with asymptotic freedom?

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From Gell-Mann—Low to ’t Hooft—Weinberg

From conceptually sound nonlinear equations for QED Green functions to technically convenient linear equations of ’t Hooft and Weinberg (present-day standard)

Scalar fields

Weinberg—scalar field is a singular case, needs extra consideration (1973)
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Veltman—MS-scheme ignores poles at dimensions $(4 - 2/\# \text{ of loops}) \equiv$ ignores quadratic divergences (1981)
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Problem 1:
Definition of the Gell-Mann–Low scheme for the most general theory

Problem 2:
Proliferation of terms in the Lagrangian after shifting the scalar field to the vacuum value (what are the evolving parameters?)
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- Specify extraction of parameters from Green functions
- Specify expression of Green functions in terms of these parameters

Generalization of gauge theories (Problem 2)
- Ignore unitarity
- Keep renormalizability
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**Inverse propagator $R$**

Let $D^{AB}(k)$ be the propagator matrix; let $R_{AB}D^{BC} = \delta_C^A$

**Inverse block propagator $\bar{R}$**

There are entries in $R$ mixing fields of different spin (corresponding to field combinations $\phi \partial_\mu A^\mu$). Suppression of these entries: $R \to \bar{R}$

**Invariant structures $T$**

$\bar{R}(k) = \sum_\alpha T_\alpha(k)r^\alpha(k^2)$

$r^\alpha(k^2)$—set of scalar “form-factors”
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**extraction of local part**

\[ \Delta_{Q^2} \bar{R}(k^2) \equiv R_{Q^2}(k^2) \]
\[ \Delta_{Q^2} \bar{R}(k) \equiv T_\alpha(k) \Delta_{Q^2} r^\alpha(k^2) \]
\[ \Delta_{Q^2} r^\alpha(k^2) = r^\alpha(Q^2) + dr^\alpha(Q^2)(k^2 - Q^2) \]

The last term with the derivative should be suppressed for the scalar form-factors from the spinor block

**locality**

\( R_{Q^2}(k) \) is a polynomial in \( k \). Its coefficients are finite parameters of the theory. \( I_{Q^2,F}(\Phi) \equiv \frac{1}{2}(R_{Q^2})^T_{AB} \Phi^A \Phi^B \) is a finite local functional. Forward reference: \( I_F(\Phi) \) is a free part of the “inaction functional”
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Extraction of parameters from three- and four-point Green functions

**extraction from three-point functions**

\[ A_3(\Phi) \rightarrow A_3(k_1, k_2, k_3) \rightarrow \sum_{\alpha} T_\alpha(k)r^\alpha(k_i k_j) \]

\( r^\alpha \) is three-index tensor; scalar in the Lorentz indexes; depending on the scalar products \( k_i k_j \)

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**Suppression of Lorentz structures**

Some structures in the complete list of Lorentz structures should be suppressed for particular models. For example:

\[ A_{\nu_1}^\mu A_{\mu \nu_2} \partial_\nu A_{\nu_3}^\nu \] should be suppressed for non-abelian gauge theories.

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\[ I_{Q^2,3}(\Phi) \text{ is a finite local functional cubic in the fields.} \]

**inaction functional** \[ I_{Q^2} \]

\[ I_{Q^2,F}(\Phi) + I_{Q^2,3}(\Phi) + I_{Q^2,4}(\Phi) \equiv I_{Q^2}(\Phi) \]

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expression of Green functions in terms of $I_{Q^2}(\Phi)$

**condition**

If the bare action of the model $S_B(\Phi)$ satisfies the condition $\Delta_{Q^2}S_B(\Phi) = S_B(\Phi)$, the connected Green functions of this model can be expressed in terms of $I_{Q^2}(\Phi)$.

**The recipe**

$$W(J) = R_{Q^2} \log T_{I_{Q^2,F}} \exp(I_{Q^2,3}(\Phi) + I_{Q^2,4}(\Phi) + iJ)$$

Here $R_{Q^2}$ is a specially tuned renormalization operation.

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Peculiarity of $R_{Q^2}$

$R_{Q^2}$ subtracts not only divergences. It also subtracts some finite parts originting from connected subgraphs (not only 1pi-subgraphs generate counterterms).

Role of combinatorics

No struggle with divergences: as soon as $\Delta_{Q^2}S_B(\Phi) = S_B(\Phi)$, $R_{Q^2}$ can be combinatorially constructed from $\Delta_{Q^2}$.

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$S_B(\Phi) \rightarrow W(J) \rightarrow I_{Q^2}(\Phi) \rightarrow S_B(\Phi)$ The inaction is the classical action with parameters extracted from Green functions (Dominicis-Englert duality for the last step)
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- No need to restrict consideration to unitary theories, if the aim is to study the renormalization group
- Renormalizability + locality + Lorentz invariance
- Example: $g f_{abc} A^a_\mu A^b_\nu (\partial^\mu A^c_\nu - \partial^\nu A^\mu_\nu) \to g_{abc}^{(3)} A^a_\mu A^b_\nu \partial^\mu A^\nu_\nu$
- Couplings are tensors with respect to gauge group and flavour indexes, like $g_{abc}^{(3)}$
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- Tensorial functions: $g^{(4)}_{abcd} = c_1 g^{(3)}_{abn} (Z^{-1})^{nm} g^{(3)}_{mcd} + ...$
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- In terms of inaction: $I^{(4)}_{Q^2} = F(I^{(3)}_{Q^2})$
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- Tensorial functions: $g_{abcd}^{(4)} = c_1 g_{abn}^{(3)} (Z^{-1})^{nm} g_{mcd}^{(3)} + ...$
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Consistency condition

- Function expressing four-point couplings in terms of three-point couplings is far from arbitrary.
- It should satisfy a consistency condition: $I_B^{(4)} = F(I_B^{(3)})$, which means that the form of this function is not renormalized.
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COUNTERTERMS TO the FOUR-POINT FUNCTIONS ARE CONSTRUCTED AS NONLINEAR COMBINATIONS OF THE COUNTERTERMS TO THREE-POINT FUNCTIONS, TWO-POINT FUNCTIONS AND TADPOLES

The possibility of this construction is a general phenomenon. The non-abelian gauge theories is a particular instance of this phenomenon.

The standard approach (renormalization as “always” and derivation of relations between renormalization constants) allows one to ignore this phenomenon.

Deriving relations between renormalization constants replaces a basic feature of the theory with a technical problem, and complicates life. Hence, the need to replace Gell-Mann–Low with ‘t Hooft-Weinberg.
Motivation

Generalization of G-L

Generalization of Gauge Theories

Example: \( g_4 \sim g_3^2 \)

Conclusions

Comments

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Potential
\[ V = \frac{\lambda}{24}(\phi^2 - v^2)^2 \]

SSB
\[ \phi = \tilde{\phi} + \tilde{v}, \langle \tilde{\phi} \rangle = 0 \]

Potential after sSB
Dropping tilde on \( \phi \)
\[ V(\phi) = \Lambda^3 \phi + \frac{m^2}{2} \phi^2 + \frac{\mu}{6} \phi^3 + \frac{\lambda}{24} \phi^4 \]

quartic coupling
\[ \lambda = \frac{\mu^2}{3m^2(1+\sqrt{1-\frac{\Lambda^3\mu}{3m^4}})} \approx \frac{\mu^2}{6m^2} + \mathcal{O}(\mu^4) \]
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Multicomponent generalization

**Tensorial structure**

The only admissible structure:

\[ \lambda_{abcd} \approx \omega \mu_{abe} (m^{-2}) ee' \mu_{e'cd} \]

**Uniqueness**

\[ \omega = \frac{1}{6} \text{ from renormalizability} \]
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Conclusions

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- Generalizations of non-abelian gauge theories are defined
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