Current Status of the Muon g-2

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1 Motivation

2 Electron g-2 and fine structure constant

3 Muon g-2 : Experiment vs Standard Model

4 Hadronic contributions to the Muon g-2
Introduction

Cosmology tell us that 95% of matter is not described in text-books yet. Dark Matter surrounds us! Where it is?

Two search strategies:
1) High energy physics to excite heavy degrees of freedom. No any evidence till now. We live in LHC era!

2) Low energy physics to produce Rare processes in view of huge statistics.

There are some rough edges of SM.

Anomalous Magnetic Moment of the Muon \((g-2)_{\mu}\) is most famous and stable (for many years) example
Dirac Equation Predicts for free point-like spin $\frac{1}{2}$ charged particle:

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ \frac{p^2}{2m} - \frac{e}{2m} (\vec{L} + 2\vec{S}) \cdot \vec{B} \right] \psi$$

$g=2, \quad a=(g-2)/2=0$ (no anomaly at tree level)

$a$ becomes nonzero due to interactions resulting in fermion substructure
1-loop QED radiative correction

\[ \Gamma_{\mu} = e\gamma_{\mu} + a_{l} \frac{ie}{2m} \sigma_{\mu\nu} q_{\nu} \]

Schwinger, 1948

\[ a = \frac{\alpha}{2\pi} = 0.001161 \]

Kush, Foley, 1948

\[ a_{\mu}^{\text{exp}} = 0.00119 \pm 0.00005 \]
Electron AMM

$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \ [0.24 \text{ ppb}] \quad \text{Harvard 2008}$$

$$a_e^{\text{SM}} = a_e^{(\text{QED})} + a_e^{(\text{hadron})} + a_e^{(\text{weak})},$$

$$a_e^{(\text{QED})} = \sum_{n=1}^{5} C_{2n} \left( \frac{\alpha}{\pi} \right)^n + \ldots$$

The theoretical error is dominated by the uncertainty in the input value of the QED coupling \( \alpha \equiv e^2/(4\pi) \)

$$a^{-1} = 137.035\,999\,1570 \ (29)(27)(18)(331)$$

Aoyama, Hayakawa, Kinoshita, Nio 2014

QED is at the level of the best theory ever built to describe nature
**Muon AMM: BNL result vs SM**

**From BNL E821 g-2 experiment (1999-2006)**

\[
a_{\mu}^{\text{BNL}} = 11,659,208.0(6.3) \times 10^{-10} (0.54 \text{ ppm})
\]

**New Exp. (2017)**

E989 at Fermilab

0.14 ppm

KEK/JParc

**In Theory**

\[
a_{\mu} = \left\{ a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{Strong}} \right\}^{\text{SM}} + ???
\]

The SM Value for \( a_{\mu} \) from \( e^+e^- \rightarrow \text{hadrons} \) (Updated 9/10)

\[
a_{\mu}^{\text{SM}} = 11,659,180.2(4.9) \times 10^{-10}
\]

**From Standard Model**

\[
\Delta a_{\mu} = a_{\mu}^{\text{BNL}} - a_{\mu}^{\text{SM}} = 27.8(8.0) \times 10^{-10} (3.6\sigma !)
\]
\[
a^\text{QED}_\mu = 11\,658\,471.8951(0.0080) \cdot 10^{-10}
\]

\[
a^\text{EW}_\mu = 15.36(0.10) \cdot 10^{-10}
\]

plus

the Hadronic Contribution estimated as

\[
a^\text{Had,LO}_\mu = 694.91(4.3) \cdot 10^{-10} \quad (<1\%\ accuracy!)
\]

\[
a^\text{Had,NLO}_\mu = -9.84(0.07) \cdot 10^{-10}
\]

\[
a^\text{Had,NNLO}_\mu = 1.24(0.01) \cdot 10^{-10}
\]

The main question how to get such accuracy from theory.
**Strong contributions to Muon AMMM**

\[
\alpha^2 \quad \alpha^3
\]

\[
HVP_{10} 
= (694.9 \pm 4.3) \cdot 10^{-10}
\]

**Hadronic Vacuum polarization**

(Davier, Hoecker, Malaescu, Zhang 2011; Hagiwara, Martin, Teubner 2011)

\[
\alpha^3
\]

\[
LbL \to g-2
\]

\[
a^\mu_{HVP} = (10.5 \pm 2.6) \cdot 10^{-10}
\]

**Hadronic Light-by-Light Scattering**

(AED, A.Radzhabov, A.Zhevlakov 11-15; C.Fischer, T. Goecke, R.Williams 11-15)

**Hadronic Vacuum Polarization**

contributes 99% and half of error

**Fixed by Experiment**

\[
a^{(2)hvp}_\mu = \frac{\alpha^2}{3\pi^2} \int ds \frac{K(s)}{s} R^{(0)}(s)
\]

**Light-by-light process**

contributes 1% and half of error

**Model Dependent**
II. Leading Order Hadronic contributions

\[ a_{\mu}^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{K(s)}{s} \]

\[ R(s) = \frac{\sigma[e^+ e^- \rightarrow \text{hadrons}]}{\sigma[e^+ e^- \rightarrow \mu^+ \mu^-]} \]

Dispersion relation, uses unitarity (optical theorem) and analyticity (Bouchiat and Michel, 1961)
HVP contributions


LO+NLO

NNLO
Hadronic light-by-light contribution to muon $g - 2$

\[ M = |e|^7 \int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4} \frac{1}{q^2 p_1^2 p_2^2 (p_4^2 - m^2)(p_5^2 - m^2)} \times \Pi_{\rho^\nu v^\alpha} (p_1, p_2, p_3) \bar{u}(p') \gamma_\alpha (p'_4 + m) \gamma_v (p'_5 + m) \gamma_\rho u(p) \]
Structure of Hadronic LbL contribution

Hierarchy in
a) \( \frac{1}{N_c} \)
b) \( M \mu / (4 \pi f_\pi ) \)
Effective Model Approach

AED, W. Broniowski PRD (08'),
AED, A. Radzhabov, A. Zhevlakov (11’—15')

\[
\mathcal{L} = \bar{q}(x)(i\gamma^\mu - m_c)q(x) + \frac{G}{2}[J^a_S(x)J^a_S(x) + J^a_P(x)J^a_P(x)] \\
- \frac{H}{4} T_{abc}[J^a_S(x)J^b_S(x)J^c_S(x) - 3J^a_S(x)J^b_P(x)J^c_P(x)], \ (1)
\]

Leading 1/Nc contribution
Nonperturbative QCD is simulated by Nonlocal Chiral Quark model

### Quark Propagator

\[
\frac{k}{k^2} \Rightarrow S(k) = \frac{k + m(k^2)}{D(k^2)}
\]

\[
k^2 \to \infty \quad \frac{k}{k^2}
\]

### Quark - Photon Vertex

\[
\gamma_\mu \Rightarrow \Gamma_\mu = \gamma_\mu + \Delta \Gamma_\mu (k, k')
\]

\[
k^2 \to \infty \quad \gamma_\mu, \text{ where } \Delta \Gamma_\mu (k, k')
\]

guarantes WTI \((k' = k + q): q_\mu \Gamma_\mu = S^{-1}(k') - S^{-1}(k)\)

### Quark - Pion Vertex

\[
\frac{1}{f_\pi} \gamma_5 \Rightarrow \Gamma_\pi = \frac{1}{f_\pi} \gamma_5 F(k, k') \quad k^2 \to \infty \quad k^2 \to \infty = 0
\]

The vertex F is equivalent of the light-cone pion WF

\[m(k^2) \text{ is related to nonlocal quark condensate and thus We use for the Dynamical Quark Mass}
\]

\[m(k^2) \approx M_q e^{-C(k^2)^a}
\]
A) Meson exchange LbL contribution

Phenomenological and QCD Constraints are used to reduce Model Dependence
Sum of $PS(\pi, \eta, \eta')$ and $S(\sigma, a_0(980), f_0(980))$ exchange contributions to $\alpha_\mu$

AED, AE Radzhabov, AS Zhevlakov (11’—14’)

$$a^{\text{LbL,PS+S}}_\mu = (6.19 \pm 0.95) \cdot 10^{-10}$$
B) Contribution of Dynamical Quark Box to $a_{\mu}$

\[
a_{m}^{Box} = \int_{0}^{\infty} dQ_1 dQ_2 \rho(Q_1, Q_2)
\]

Aldins, Brodsky, Dufner, Kinoshita 1970

$Q_3 = -Q_2 - Q_1$

$Q_4 \to 0,$
1) Monotonously decreasing at large Q
2) Rapidly growing at small Q

\[ a_\mu = \int dQ_1 dQ_2 \rho(Q_1, Q_2) \]
1) Completely different structure

2) Changes sign at small $Q$

3) Island with Hill appears
1) Becomes more regular

2) Island with Hill grows
1) Becomes more regular

2) Island with Hill grows
1) Completely Regular

2) With clear Island and Hill

\[ a_{mu}^{Box} = \int_{0}^{\infty} dQ_1 dQ_2 \rho(Q_1, Q_2) \]
1) LET at $Q=0$
2) $p$QCD at large $Q$

This is due to Gauge Invariance and Spontaneous breaking of Chiral Symmetry
Estimates of Hadronic Contributions in different Approaches

<table>
<thead>
<tr>
<th>Model</th>
<th>$\pi^0$</th>
<th>PS ($\pi^0, \eta, \eta'$)</th>
<th>S ($\sigma, f_0, a_0$)</th>
<th>AV</th>
<th>Quark loop</th>
<th>$\pi, K-$ loops</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>VMD (Hayakawa [24])</td>
<td>5.74(0.36)</td>
<td>8.27(0.64)</td>
<td>-0.68(0.2)</td>
<td>0.17(0.10)</td>
<td>0.97(1.11)</td>
<td>-0.45(0.81)</td>
<td>8.96(1.54)</td>
</tr>
<tr>
<td>ENJL (Bijnens [25])</td>
<td>5.58(0.05)</td>
<td>8.5(1.3)</td>
<td>-0.25(0.1)</td>
<td>2.1(0.3)</td>
<td></td>
<td>-1.9(1.3)</td>
<td>8.3(3.2)</td>
</tr>
<tr>
<td>LMD+V (Knecht [26])</td>
<td>5.8(1.0)</td>
<td>8.3(1.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.0(4.0)</td>
</tr>
<tr>
<td>Q-box (Pivovarov [32])</td>
<td>7.65(1.0)</td>
<td>11.4(1.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.05</td>
</tr>
<tr>
<td>LENJL (Bartos [31])</td>
<td>8.18(1.65)</td>
<td>9.55(1.7)</td>
<td>1.23(0.24)</td>
<td>2.2(0.5)</td>
<td></td>
<td></td>
<td>14.05</td>
</tr>
<tr>
<td>(LMD+V)'(Melnikov [27])</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.77(1.68)</td>
</tr>
<tr>
<td>NχQM (Dorokhov [36–38])</td>
<td>5.01(0.37)</td>
<td>5.85(0.87)</td>
<td>0.34(0.48)</td>
<td>11.0(0.9)</td>
<td></td>
<td></td>
<td>16.8(1.25)</td>
</tr>
<tr>
<td>oLMDV (Nyffeler [28])</td>
<td>7.2(1.2)</td>
<td>9.9(1.6)</td>
<td>-0.7(0.2)</td>
<td>2.2(0.5)</td>
<td></td>
<td>-1.9(1.3)</td>
<td>11.6(0.4)</td>
</tr>
<tr>
<td>DS (Goecke [39])</td>
<td>5.75(0.69)</td>
<td>8.07(1.2)</td>
<td></td>
<td>2.1(0.3)</td>
<td></td>
<td>10.7(0.2)</td>
<td>18.8(0.4)</td>
</tr>
<tr>
<td>CχQM (Greynat [35])</td>
<td>6.8(0.3)</td>
<td>6.8(0.3)</td>
<td></td>
<td>8.2(0.6)</td>
<td></td>
<td></td>
<td>15.0(0.3)</td>
</tr>
</tbody>
</table>
Parameters fitted by Pion Mass and its 2-gamma decay, k0 mass and Eta to 2gamma decay
Comparison with some recent calculations

(Goeke, Fischer, Williams)
(Greynat, de Rafael)

Our results
Our results indicate that the HLbL is underestimated and discrepancy may be less than 3 sigma

\[ \Delta a_\mu \cdot 10^{+10} = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 27.8(8.0) \Rightarrow \approx 23 \]
Summary

1) Study of Electron AMM provides very precise value for the QED coupling $\alpha$

2) Study of Muon AMM is sensitive to effects of SM and NP

3) At present there is $3.4\sigma$ disagreement between SM and BNL experiment. New experiments at FNAL and Jparc are promising

4) New experiments at VEPP2000, KLOE2, BESS III on cross section will further diminish the error for HVP contribution

5) The account of full kinematic dependence of meson-two-photon vertex reduces the value for the meson exchange LbL contribution

6) Dynamical quark box contribution make total result bigger than in previous estimates
Feynman rules in Nonlocal Effective Models

\[ ie \Gamma_{\mu}^{NL}(p_1, q, p_2) = ie Q \gamma_{\mu} - e Q(p_1 + p_2) \frac{\Sigma(p_2) - \Sigma(p_1)}{p_2^2 - p_1^2} \]

\[ -iq_{\mu}^{\nu} \Gamma_{\mu}^{NL}(p_1, q, p_2) = S^{-1}(p_2)Q - QS^{-1}(p_1) \]

\[ S^{-1}(p) = i \not{p} + \Sigma(p) \]

Ward-Takahashi Identity

\[ g^2 \Gamma^{\mu a, \nu b}(p, q_1, q_2, p + q_1 + q_2) \]

\[ = -g^2 \left[ \frac{(T^a T^b + T^b T^a) g^{\mu \nu}}{2p \cdot (q_1 + q_2) + (q_1 + q_2)^2} \left[ \Sigma(p + q_1 + q_2) - \Sigma(p) \right] \right. \]
\[ + T^a T^b \left. \frac{(2p + q_2)^{\mu} [2(p + q_2) + q_1]^{\nu}}{2(p + q_2) \cdot q_1 + q_1^2} \left[ \frac{\Sigma(p + q_1 + q_2) - \Sigma(p)}{2p \cdot (q_1 + q_2) + (q_1 + q_2)^2} - \frac{\Sigma(p + q_2) - \Sigma(p)}{2p \cdot q_2 + q_2^2} \right] \right. \]
\[ + T^b T^a \left. \frac{(2p + q_1)^{\mu} [2(p + q_1) + q_2]^{\nu}}{2(p + q_1) \cdot q_2 + q_2^2} \left[ \frac{\Sigma(p + q_1 + q_2) - \Sigma(p)}{2p \cdot (q_1 + q_2) + (q_1 + q_2)^2} - \frac{\Sigma(p + q_1) - \Sigma(p)}{2p \cdot q_1 + q_1^2} \right] \right. \]

\[ \left. \left. \right. \right] \]
LIFE OF A MUON: THE g-2 EXPERIMENT

Protons from AGS.

Hit Target.

Pions, weighing 1/6 proton, are created.

Pions decay to muons.

Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.

After each circle, muon's spin axis changes by 12°, yet it keeps on traveling in the same direction.

One of 24 detectors see an electron, giving the muon spin direction; g-2 is this angle, divided by the magnetic field the muon is traveling through in the ring.

After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.
Precise measurement of muon $g-2/EDM$ at JPARC