Pion polarizabilities:
Theory vs. Experiment

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Hadron Structure 2015
Pion polarizabilities: definition

Transverse gauge $\epsilon_i = (0, \vec{\epsilon}_i)$

LAB frame $p_1 = (M_\pi, \vec{0})$

Expansion in $q_i = (\omega_i, \vec{q}_i)$:
Pion polarizabilities: definition

Transverse gauge $\epsilon_i = (0, \vec{\epsilon}_i)$

LAB frame $p_1 = (M_\pi, \vec{0})$

Expansion in $q_i = (\omega_i, \vec{q}_i)$:

$$T_{\gamma \pi^+ \rightarrow \gamma \pi^+} = \begin{cases} \begin{aligned} & -2 e^2 \vec{\epsilon}_1 \cdot \vec{\epsilon}_2 \\ & \text{Born term} \end{aligned} \end{cases}$$

$$+ 8 \pi M_\pi \left\{ \alpha_\pi \omega_1 \omega_2 \vec{\epsilon}_1 \cdot \vec{\epsilon}_2 + \beta_\pi (\vec{\epsilon}_1 \times \vec{q}_1) \cdot (\vec{\epsilon}_2 \times \vec{q}_2) \right\} + \ldots$$

el-mag polarizabilities
The electric, $\alpha_\pi$, and magnetic, $\beta_\pi$, polarizabilities characterize the response of hadrons to their two-photon interactions at low energies. These quantities are analogous to electromagnetic radii and magnetic moments which characterize the response of hadrons to their single-photon interactions at low energies.

\[
q = p_1 - p_2
\]

\[
< \pi(p_2)|j^\mu|\pi(p_1) > = e(p_1 + p_2)^\mu F_\pi(q^2)
\]

\[
F_\pi(q^2) = 1 + \frac{1}{6} r_\pi^2 q^2 + \ldots
\]
The concept of the polarizability of molecules, atoms and nuclei was applied for the first time to hadrons in

A. Klein, Phys. Rev. 99 (1955) 998,
A.M. Baldin, Nucl. Phys. 18 (1960) 310,
V.A. Petrun’kin, JETP 13 (1961) 804

Many theoretical papers afterwards

Only a few experiments
The units of measurement

As follows from the definition, the dipole pion polarizabilities are proportional to

\[ \alpha_{\pi}(\beta_{\pi}) \sim \frac{\alpha}{M_{\pi}} \frac{1}{\Lambda^2} \approx 4 \times 10^{-4} \text{ fm}^3 \]

where the hadronic scale \( \Lambda \sim 4\pi F_{\pi} \sim 1 \text{ GeV} \).

Then a natural choice of units for the dipole polarizabilities is \( 10^{-4} \text{ fm}^3 \).
(a) The scattering of high energy pions off the Coulomb field of heavy nucleus.

(b) The radiative pion photoproduction from the proton.

(c) The pion pair production in photon-photon collisions.
GIS'06 = Gasser, Ivanov, Sainio, Nucl. Phys. B745 (2006) 84
General properties of pion polarizabilities

- Classical sum rule (Petrun’kin’64):

\[
\alpha_\pi = \frac{\alpha}{3m} < r^2_\pi > + 2\alpha \sum_{n \neq 0} \left| \frac{< n|\mathcal{D}|0 >}{E_n - E_0} \right|^2
\]

where \( \mathcal{D} \) is the electric dipole operator.

\[\alpha_{\pi \pm} \mapsto (3.5 - 6.8)\]

- The optical theorem relates the sum of polarizabilities to an unsubtracted forward dispersion relation

\[(\alpha + \beta)_\pi = \frac{M^2_\pi}{\pi^2} \int_0^\infty \frac{ds'}{4M^2_\pi} \frac{\sigma_{\gamma\pi}^{\text{tot}}(s')}{(s' - M^2_\pi)^2} > 0\]
Using current algebra/PCAC gives the relation of $\alpha_\pi(\beta_\pi)$ with the vector $F_V$ and axial $F_A$ structure constants for radiative pion decays $\pi^- \rightarrow e^+\nu\gamma$

(Terent’ev’73):

$$\alpha_{\pi\pm} = -\beta_{\pi\pm} = \frac{\alpha}{8\pi^2 F_\pi^2 M_\pi} \frac{F_A}{F_V} = 2.7 \pm 0.4$$

Models

- Quark loop diagrams (Gerasimov’1979) \[ \alpha_{\pi^\pm} \approx 6 \]

- Nonlinear $\sigma$-model, chiral quark loops, NJL-model (Volkov, Pervushin et al.’1975, ...) \[ \alpha_{\pi^\pm} \mapsto (5.0 - 5.8) \]

- Electric polarizability of neutral pions in NJL model
  Ahmadov, Kuraev, Volkov’ 2011 \[ |\alpha_{\pi^0}| \approx 0.5 \]

- Magnetic polarizability of neutral pions from lattice
  Luschevsckaya, . . . , Teryaev’ 2014 \[ \beta_{\pi^0} = 2.75 \pm 1.31 \]

- Nonlocal chiral quark model (Dorokhov, Broniowski’2003) \[ \alpha_{\pi^\pm} \approx 2.9 \]

- Quark confinement model (Efimov, Ivanov, Mizutani’1992) \[ \alpha_{\pi^\pm} \approx 3.6 \]
Effective Lagrangians: $\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{\text{eff}}$ for $E \ll M_\rho$

Weinberg’1979; Leutwyler’1994

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

$\mathcal{L}_{\text{eff}}$ expressed in observed hadron fields, has the same symmetry as QCD.

- The leading order in chiral SU(2) (pions and photons only):

  $$\mathcal{L}_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + M^2 (U + U^\dagger) \rangle,$$

  $$D_\mu U = \partial_\mu U - i(QU - UQ)A_\mu,$$

  $U \in SU(2)$, contains the pion fields

- $M^2 = (m_u + m_d)B$

- $F, B$ are low-energy constants (LECs) not fixed by chiral symmetry

- $\mathcal{L}_2$ - nonrenormalizable quantum field theory
Higher order Lagrangians

\[ \mathcal{L}_4 = \sum_{i=1}^{10} \ell_i K_i = \ell_1 \frac{1}{4} \langle D_\mu U D^\mu U^\dagger \rangle^2 + \cdots , \]

\[ \mathcal{L}_6 = \sum_{i=1}^{57} c_i P_i, \quad (57 \rightarrow 56) \quad \text{Haefeli, Ivanov, Schmid, Ecker 2007} \]

Local monomials \( K_i, P_i \) are known

Gasser, Leutwyler 1984,1985; Bijnens, Colangelo, Ecker 1999

LECs \( \ell_i, c_i \) absorb the divergences at order \( p^4 \) and \( p^6 \)

Notation later on: \( \ell_i, c_i \rightarrow \ell_i^r, c_i^r \) UV finite parts of \( \ell_i, c_i; \)
\( \ell_i^r, c_i^r \rightarrow \ell_i, c_i \) scale independent parts of \( \ell_i, c_i. \)
Higher order Lagrangians

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Calculations with \( \mathcal{L}_{\text{eff}} \) give an expansion in quark masses and external momenta.

Chiral perturbation theory (ChPT)

\[ \quad \text{Gasser, Leutwyler 1984,1985} \]
Pion polarizabilities in ChPT to one-loop

- Chiral expansion to one-loop
  
  \[ \alpha_\pi = -\beta_\pi = \frac{\alpha}{8\pi^2 F^2_\pi M_\pi} \cdot \frac{1}{6} (\bar{\ell}_6 - \bar{\ell}_5) \]

- The LECs \( \bar{\ell}_{5,6} \) also arise in \( \pi \rightarrow e\nu\gamma \)-amplitude

- This gives a low energy theorem
  
  \[ \alpha_\pi = -\beta_\pi = \frac{\alpha}{8\pi^2 F^2_\pi M_\pi} \frac{F_A}{F_V} \left\{ 1 + O(M_\pi^2) \right\} \]

Bijnens, Cornet 1988
Donoghue, Holstein 1989
Terent’ev 1973
Pion polarizabilities in ChPT to two-loop

Gasser, Ivanov, Sainio 2005, 2006

Diagram representations of pion polarizabilities.
Invoke a dispersion relation for the function

\[ I(\mu, n; s) = \int_{0}^{1} dx \left[ x (1 - x)\right]^n \left[ 1 - s x (1 - x)\right]^{\mu} \]

\[ = \int_{4}^{\infty} d\sigma \frac{\rho(\mu, n; \sigma)}{\sigma - s} \quad (-1 < \mu < 0) \]
Cross section $\gamma\gamma \rightarrow \pi^+\pi^-$

$|\cos\theta|<0.6$

Experimental data from MARK II (SLAC) 1990
Charged pion polarizabilities: analytic results

\[(\alpha_1 \pm \beta_1)_{\pi^+} = \frac{\alpha}{16 \pi^2 F_{\pi}^2 M_{\pi}} \left\{ c_{1\pm} + \frac{M_{\pi}^2}{16 \pi^2 F_{\pi}^2} d_{1\pm} + O(M_{\pi}^4) \right\}, \]

\[c_{1+} = 0, \quad c_{1-} = \frac{2}{3} \bar{\ell}_\Delta, \]

\[d_{1+} = 8 b^r - \frac{4}{9} \left\{ \ell \left( \ell + \frac{1}{2} \bar{\ell}_1 + \frac{3}{2} \bar{\ell}_2 \right) - \frac{53}{24} \ell + \frac{1}{2} \bar{\ell}_1 + \frac{3}{2} \bar{\ell}_2 + \frac{91}{72} + \Delta_+ \right\} \]

\[d_{1-} = a^r_1 + 8 b^r - \frac{4}{3} \left\{ \ell \left( \bar{\ell}_1 - \bar{\ell}_2 + \bar{\ell}_\Delta - \frac{65}{12} \right) - \frac{1}{3} \bar{\ell}_1 - \frac{1}{3} \bar{\ell}_2 + \frac{1}{4} \bar{\ell}_3 - \bar{\ell}_\Delta \bar{\ell}_4 + \frac{187}{108} + \Delta_- \right\} \]
Charged pion polarizabilities: analytic results

\[
(\alpha_1 \pm \beta_1)_{\pi^+} = \frac{\alpha}{16 \pi^2 F^2_F M_\pi} \left\{ c_{1\pm} + \frac{M^2_\pi}{16 \pi^2 F^2_F} d_{1\pm} + O(M^4_\pi) \right\},
\]

\[
c_{1+} = 0, \quad c_{1-} = \frac{2}{3} \bar{\ell}_\Delta,
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\]

\[
d_{1-} = a'_r + 8 b^r - \frac{4}{3} \left\{ \ell \left( \bar{\ell}_1 - \bar{\ell}_2 + \bar{\ell}_\Delta - \frac{65}{12} \right) - \frac{1}{3} \bar{\ell}_1 - \frac{1}{3} \bar{\ell}_2 + \frac{1}{4} \bar{\ell}_3 - \bar{\ell}_\Delta \bar{\ell}_4
\]

\[
\Delta_+ = \frac{8105}{576} - \frac{135}{64} \pi^2 = -6.75,
\]

\[
\Delta_- = \frac{41}{432} - \frac{53}{64} \pi^2 = -8.08.
\]

Burgi: \(-8.69\) \quad Burgi: \(-8.73\)
Numerical values of LECs

\[ \bar{\ell}_1 = -0.4 \pm 0.6, \quad \bar{\ell}_2 = 4.3 \pm 0.1, \quad \bar{\ell}_3 = 2.9 \pm 2.4, \quad \bar{\ell}_4 = 4.4 \pm 0.2 \]

Colangelo, Gasser, Leutwyler 2001

\[ \bar{\ell}_\Delta \doteq \bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3. \]

Bijnens, Talavera 1997
Numerics

Numerical values of LECs

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\[ \bar{\ell}_\Delta \equiv \bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3. \]

Bijnens, Talavera 1997

\[
\begin{align*}
  a'_1 &= -4096\pi^4 \left( 6c'_6 + c'_r_{29} - c'_r_{30} - 3c'_r_{34} + c'_r_{35} + 2c'_r_{46} - 4c'_r_{47} + c'_r_{50} \right) \\
  b' &= -128\pi^4 \left( c'_r_{31} + c'_r_{32} - 2c'_r_{33} - 4c'_r_{44} \right)
\end{align*}
\]

Resonance \( \rho, a_1, b_1 \) exchange at \( \mu = M_\rho \)

\( (a'_1, a'_2, b') = (-3.2, 0.7, 0.4) \)

ENJL model with large \( N_c \) (Bijnens & Prades)

\( (a'_1, a'_2, b') = (-8.7, 5.9, 0.38) \)

We use \( b' = 0.4 \pm 0.4 \) and vary \( a'_1 \) from -10 to 0.
Chiral expansion at the Compton threshold

Example: spin non-flip amplitude. Solid line $\rightarrow$ two-loops, dashed line $\rightarrow$ one-loop
Charged pion polarizabilities


<table>
<thead>
<tr>
<th></th>
<th>ChPT to one loop</th>
<th>ChPT to two-loops</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\alpha - \beta)_\pi^+$</td>
<td>$6.0 \pm 0.6$</td>
<td>$5.7 \pm 1.0$</td>
</tr>
<tr>
<td>$(\alpha + \beta)_\pi^+$</td>
<td>$0$</td>
<td>$0.16 \pm 0.14$</td>
</tr>
</tbody>
</table>
Proposal to measure pion polarizability via Primakoff reaction

A.G. Galperin, G.V. Mitselmakher, A.G. Olshevski and V.N. Pervushin

Yad. Fiz. 32 (1980) 1053

- The first observation of the Compton scattering off pion at SIGMA spectrometer.

- The first measurement of pion polarizabilities

- Dubna group brought their experience to the COMPASS experiment
Main advantages of COMPASS

- One can use pion and muon beams of the same momentum with the same setup configuration.

\[ \pi^- (A, Z) \rightarrow \pi^- (A, Z) \gamma \]
\[ \mu^- (A, Z) \rightarrow \mu^- (A, Z) \gamma \]

- Muon is the point-like particle and corresponding cross section for muon is known with high precision.

- So, muon data can be used as reference to control the systematics.
Primakoff reaction:
\[ \pi^- + Z \rightarrow \pi^- + Z + \gamma \]

\[ E_{\pi} = 190 \text{ GeV}, \; Q^2 < 0.0015 \text{ GeV}^2 \]

63 \cdot 10^3 \text{ events}

\[ \alpha_{\pi} = 2.0 \pm 0.6 \text{ (stat)} \pm 0.7 \text{ (syst)} \]

assumption: \( \alpha_\pi + \beta_\pi = 0 \)
Plot: B. Badelek (COMPASS) 2015

### Experimental information

<table>
<thead>
<tr>
<th>Experiments</th>
<th>$(\alpha - \beta)_\pi^{\pm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma p \rightarrow \gamma \pi^+ n$ Mainz (2005)</td>
<td>$11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{mod}}$</td>
</tr>
<tr>
<td>L. Fil’kov, V. Kashevarov (2005) $\gamma\gamma \rightarrow \pi^+\pi^-$ available data</td>
<td>$13.0^{+2.6}_{-1.9}$</td>
</tr>
<tr>
<td>A. Kaloshin, V. Serebryakov (1994) $\gamma\gamma \rightarrow \pi^+\pi^-$ MARK II</td>
<td>$5.25 \pm 0.95$</td>
</tr>
<tr>
<td>J.F. Donoghue, B. Holstein (1993) $\gamma\gamma \rightarrow \pi^+\pi^-$ MARK II</td>
<td>$5.4$</td>
</tr>
<tr>
<td>D. Babusci et al. (1992) $\gamma\gamma \rightarrow \pi^+\pi^-$ PLUTO DM 1</td>
<td>$38.2 \pm 9.6 \pm 11.4$</td>
</tr>
<tr>
<td>DM 2</td>
<td>$34.4 \pm 9.2$</td>
</tr>
<tr>
<td>MARK II</td>
<td>$52.6 \pm 14.8$</td>
</tr>
<tr>
<td>$4.4 \pm 3.2$</td>
<td></td>
</tr>
<tr>
<td>$\gamma p \rightarrow \gamma \pi^+ n$ Lebedev Inst. (1986)</td>
<td>$40 \pm 24$</td>
</tr>
<tr>
<td>$\pi^- Z \rightarrow \gamma \pi^- Z$ Serpukhov (1983) COMPASS (2015)</td>
<td>$15.6 \pm 6.4_{\text{stat}} \pm 4.4_{\text{syst}}$</td>
</tr>
<tr>
<td></td>
<td>$4.0 \pm 1.2_{\text{stat}} \pm 1.4_{\text{syst}}$</td>
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Summary

- ChPT is a successful tool to analyse low-energy physics.
- Chiral expansion for the $\gamma\gamma \rightarrow \pi\pi$ amplitude at the Compton threshold converges quite rapidly.
- Two-loop result for the charged pion polarizability $(\alpha - \beta)_\pi$ is in agreement with very well known low-energy theorem.
- However, there is a clash almost a factor of 2 (!) between this result and several experiments.
- The last precise measurement of $\alpha_{\pi^+}$ performed by COMPASS is found in agreement with ChPT.
- Another experiments in CERN with JINR participation aiming to check the ChPT-predictions:
  - **DIRAC**: $\pi\pi$-scattering lengths from $\pi^+\pi^-$ – atom
  - **NA48**: $\pi\pi$-scattering lengths from $K \rightarrow 3\pi$ – cusp