Nurgul Habyl (Almaty, Kazakhstan)

The semileptonic decay $\Lambda_b \rightarrow \Lambda_c + l^- + \bar{\nu}_l$
in the covariant confined quark model

SLOVAKIA, HADRON STRUCTURE 2015

Th. Gutsche, M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij, P. Santorelli, N. Habyl

*Physical Review D 91, 074001 (2015)*
Contents

• Introduction
• Matrix element and helicity amplitudes for $\Lambda_b \rightarrow \Lambda_c + l^- + \bar{\nu}_l$
• Physical observables
• Covariant confined quark model
• Numerical results
• Conclusion
Introduction

• Recently there has been much discussion about tensions of some of the experimental results on leptonic, semileptonic and rare decays involving $\mu$ and $\tau$ leptons with the predictions of the Standard Model (SM)

$$R(D) \equiv \frac{Br(\bar{B} \to D \tau \bar{\nu}_\tau)}{Br(\bar{B} \to D l \bar{\nu}_l)} \quad (D = D, D^*; \ l = e, \mu)$$

$$R(D) = \begin{cases} 0.305 \pm 0.012 & SM \\ 0.421 \pm 0.058 & Babar & Belle \end{cases} \quad R(D^*) = \begin{cases} 0.252 \pm 0.004 & SM \\ 0.337 \pm 0.025 & Babar & Belle \end{cases}$$

• This observation has inspired a number of searches for new physics beyond the SM (BSM) in charged current interactions.
Introduction

• Motivated by the discrepancy between theory and experiment in the meson sector, we analyze the corresponding semileptonic baryon decays \( \Lambda_b^0 \rightarrow \Lambda_c^+ + \tau^- + \bar{\nu}_\tau \) within the SM.

• We calculate the total rate, the differential decay distributions, the longitudinal and transverse polarization of the daughter baryon and lepton-side forward-backward asymmetries.

• We provide numerical results on these observables using the covariant confined quark model.
Matrix element and helicity amplitudes

• The transition matrix element for the process $\Lambda_b \rightarrow \Lambda_c^+ + l^- + \bar{\nu}_l$ can be written as:

$$M(\Lambda_{b[ud]} \rightarrow \Lambda_{c[ud]} l^- \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{bc} \left\langle \Lambda_{c[ud]} \left| \bar{c} O^\mu b \right| \Lambda_{b[ud]} \right\rangle (l^- O_\mu \bar{\nu}_l)$$

where $O^\mu = \gamma^\mu (1 - \gamma_5)$, $G_F$ is Fermi coupling constant, $V_{bc}$ is the Cabibbo-Kobayashi-Maskawa matrix element.

• This process is described in our model by the diagram

![Diagram](image-url)
Matrix element and helicity amplitudes

- Hadronic matrix element \( \langle \Lambda_{c[ud]} | \bar{c} O^\mu b | \Lambda_{b[ud]} \rangle \) can be expressed via form factors \( F_i^J \) \( (i = 1, 2, 3; \quad J = V, A) \)

\[
M^V_\mu (\lambda_1, \lambda_2) = \langle \Lambda_c, \lambda_2 | J^V_\mu | \Lambda_b, \lambda_1 \rangle = \bar{u}_2(p_2, \lambda_2) \times \\
\left[ F_1^V(q^2)\gamma_\mu - \frac{F_2^V(q^2)}{M_1} i\sigma_{\mu\nu}q^\nu + \frac{F_3^V(q^2)}{M_1} q_\mu \right] u_1(p_1, \lambda_1),
\]

\[
M^A_\mu (\lambda_1, \lambda_2) = \langle \Lambda_c, \lambda_2 | J^A_\mu | \Lambda_b, \lambda_1 \rangle = \bar{u}_2(p_2, \lambda_2) \times \\
\left[ F_1^A(q^2)\gamma_\mu - \frac{F_2^A(q^2)}{M_1} i\sigma_{\mu\nu}q^\nu + \frac{F_3^A(q^2)}{M_1} q_\mu \right] \gamma_5 u_1(p_1, \lambda_1),
\]

where \( \sigma_{\mu\nu} = \frac{i}{2} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \), \( q = p_1 - p_2 \).

The labels \( \lambda_i = \pm \frac{1}{2} \) denote the helicities of the two baryons.
Matrix element and helicity amplitudes

- We calculate the helicity amplitudes in the rest frame of the parent baryon $\Lambda_b$ where we choose the $z$–axis to be along the $W_{off-shell}^{-}$.

- Helicity amplitudes are written as

$$H_{\lambda_2W}^{V/A} = M_{\mu}^{V/A}(\lambda_2) \in \mu (\lambda_W)$$

where there are four helicities for the $W_{off-shell}^{-}$ with two $J=1,0$ angular momentum of the rest frame $W_{off-shell}^{-}$.\[\lambda_W = \pm 1, 0 \ (J=1) \quad \text{and} \quad \lambda_W = 0, \ (J=0)\]

- We adopt the notation $\lambda_W = 0$ for $J=1$, $\lambda_W = t$ for $J=0$

- From angular momentum conservation one has $\lambda_1 = \lambda_2 - \lambda_W$
Matrix element and helicity amplitudes

• They read

\[
H_{V/A}^{+1/2} = \sqrt{Q_{\pm}} \left( M_m F_1^{V/A} \pm \frac{q^2}{M_1} F_3^{V/A} \right)
\]

\[
H_{V/A}^{+1/0} = \sqrt{2Q_{\pm}} \left( F_1^{V/A} \pm \frac{M_{\pm}}{M_1} F_2^{V/A} \right)
\]

where we make use of the abbreviations \( M_{\pm} = M_1 \pm M_2 \), \( Q_{\pm} = M_{\pm}^2 - q^2 \)

• The total left–chiral helicity amplitude is defined by the composition

\[
H_{\lambda_2,\lambda_w} = H_{\lambda_2,\lambda_w}^V - H_{\lambda_2,\lambda_w}^A
\]

• Bilinear combinations of helicity amplitudes gives a helicity structure functions, which will be used to calculate the decay rate.
• Definition of helicity structure functions and their parity properties

<table>
<thead>
<tr>
<th>Parity-conserving</th>
<th>Parity-violating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{t'} =</td>
<td>H^{1/2}<em>{+1/2} + H^{-1/2}</em>{-1/2}</td>
</tr>
<tr>
<td>$H_L =</td>
<td>H^{1/2}<em>{+1/2} + H^{-1/2}</em>{-1/2}</td>
</tr>
<tr>
<td>$H_S =</td>
<td>H^{1/2}<em>{+1/2} - H^{-1/2}</em>{-1/2}</td>
</tr>
<tr>
<td>$H_{LT} = \text{Re} \left( H^{1/2}<em>{+1/2} H^\dagger</em>{-1/2} + H^{1/2}<em>{-1/2} H^\dagger</em>{+1/2} \right)$</td>
<td>$H_{LTP} = \text{Re} \left( H^{1/2}<em>{+1/2} H^\dagger</em>{-1/2} - H^{1/2}<em>{-1/2} H^\dagger</em>{+1/2} \right)$</td>
</tr>
<tr>
<td>$H_{ST} = \text{Re} \left( H^{1/2}<em>{+1/2} H^\dagger</em>{+1/2} + H^{1/2}<em>{-1/2} H^\dagger</em>{-1/2} \right)$</td>
<td>$H_{STP} = \text{Re} \left( H^{1/2}<em>{+1/2} H^\dagger</em>{+1/2} - H^{1/2}<em>{-1/2} H^\dagger</em>{-1/2} \right)$</td>
</tr>
<tr>
<td>$H_{SL} = \text{Re} \left( H^{1/2}<em>{+1/2} H^\dagger</em>{+1/2} + H^{1/2}<em>{-1/2} H^\dagger</em>{-1/2} \right)$</td>
<td>$H_{SLP} = \text{Re} \left( H^{1/2}<em>{+1/2} H^\dagger</em>{+1/2} - H^{1/2}<em>{-1/2} H^\dagger</em>{-1/2} \right)$</td>
</tr>
</tbody>
</table>
The angular decay distribution for
\[ \Lambda_b(p_1) \rightarrow \Lambda_c^+(p_2) + W_{\text{off shell}} \rightarrow l^-(p_l) + \bar{\nu}_l(p_{\nu_l}) \]

- The differential distribution

\[
\frac{d\Gamma}{dq^2d\cos\theta} = \frac{G_F^2}{(2\pi)^4} |V_{bc}|^2 \frac{(q^2 - m_i^2) |p_2|}{128M_i^2 q^2} H^{\mu\nu} L_{\mu\nu}(\theta)
\]

here \( q = p_l + p_{\nu_l} \), \( p_i^2 = m_i^2 \), \( p_{\nu_i}^2 = 0 \) and \( |p_2| = \lambda^{1/2}(M_1^2, M_2^2, q^2) / (2M_1) \).

- One find the hadron and lepton tensors as follows

\[
H^{\mu\nu} L_{\mu\nu}(\theta) = \frac{16\pi}{3} (2q^2 \nu) W(\theta)
\]

\[
W(\theta) = \frac{3}{8} (1 + \cos^2\theta) H_U - \frac{3}{4} \cos\theta H_P + \frac{3}{4} \sin^2\theta H_L \\
+ \delta_l \left\{ \frac{3}{2} H_S + \frac{3}{4} \sin^2\theta H_U + \frac{3}{2} \cos^2\theta H_L - 3\cos\theta H_{SL} \right\}, \quad \delta_l = \frac{m_i^2}{2q^2}.
\]
The angular decay distribution for
\[ \Lambda_b(p_1) \rightarrow \Lambda_c^+(p_2) + W_{\text{off shell}} \rightarrow l^-(p_l) + \bar{\nu}_l(p_{\nu_l}) \]

• Differential rate for the three body decay
\[ \frac{d\Gamma}{dq^2} = \Gamma_0 \frac{(q^2 - m_i^2)^2 |p_2|}{M_1^7 q^2} \{H_U + H_L + \delta_l[H_U + H_L + 3H_S]\} = \Gamma_0 \frac{(q^2 - m_i^2)^2 |p_2|}{M_1^7 q^2} H_{\text{tot}} \]

\[ H_{\text{tot}} = \int d\cos \theta W(\theta) = \{H_U + H_L + \delta_l[H_U + H_L + 3H_S]\}, \quad \Gamma_0 = \frac{G^2 |V_{bc}|^2 M_1^5}{192\pi^3} \]

• It is convenient to define partial rates \( \frac{d\Gamma_x}{dq^2} \) and \( \frac{d\tilde{\Gamma}_x}{dq^2} \) for the helicity-nonflip and helicity-flip helicity structure functions \( H_x \)

\[ \frac{d\Gamma_x}{dq^2} (nf) = \Gamma_0 \frac{(q^2 - m_i^2)^2 |p_2|}{M_1^7 q^2} H_x, \quad X = U, L, P, L_p, LT, LT_p \]

\[ \frac{d\tilde{\Gamma}_x}{dq^2} (hf) = \delta \Gamma_0 \frac{(q^2 - m_i^2)^2 |p_2|}{M_1^7 q^2} H_x, \quad X = U, L, LT, P, SL, S, L_p, S_p, SL_p, LT_p, ST_p \]

\[ \frac{d\Gamma_x}{dq^2} = \frac{d\Gamma_x}{dq^2} (nf) + \frac{d\tilde{\Gamma}_x}{dq^2} (hf) \]
Polarization of the daughter baryon $\Lambda_c$ and the charged lepton $l^-$

- One can define a lepton-side forward-backward asymmetry defined by

$$A'_{FB}(q^2) = \frac{d\Gamma(F) - d\Gamma(B)}{d\Gamma(F) + d\Gamma(B)} = -\frac{3}{2} \frac{H_p + 4\delta_l H_{SL}}{H_{tot}}$$

where $d\Gamma(F)$ and $d\Gamma(B)$ denote the rates in the forward and backward hemispheres.

- A convexity parameter $C_F(q^2)$ is defined as

$$C_F(q^2) = \frac{3}{4} (1 - 2\delta_l) \frac{H_U + 2H_L}{H_{tot}}$$
Polarization of the daughter baryon $\Lambda_c$ and the charged lepton $l^-$

- Longitudinal and transverse polarization components of the daughter baryon are defined as

$$ P_z^h \left( q^2 \right) = \frac{H_P + H_{L_P} + \delta_l (H_P + H_{L_P} + H_{S_P})}{H_{tot}}, \quad P_x^h \left( q^2 \right) = -\frac{3\pi}{4\sqrt{2}} \frac{H_{LT} - 2\delta_l H_{ST_P}}{H_{tot}} $$

- Longitudinal and transverse polarization component of the charged lepton read

$$ P_z^l \left( q^2 \right) = -\frac{H_U + H_L - \delta_l (H_U + H_L + 3H_S)}{H_{tot}} $$

$$ P_x^l \left( q^2 \right) = -\frac{3\pi}{4\sqrt{2}} \sqrt{\delta_l} \frac{P - 2H_{SL}}{H_{tot}} $$
Four fold angular decay distribution

\[
\frac{d\Gamma(\Lambda_b \to \Lambda_c^+ (\to \Lambda \pi) + l\bar{\nu}_l)}{dq^2d \cos \theta d \chi d \cos \theta_B} = \frac{1}{2} \frac{1}{2\pi} \frac{Br(\Lambda_c^+ \to \Lambda + \pi^+)}{\Gamma_0} \frac{(q^2 - m_l^2)^2}{M_1^7 q^2} |p_2| \times
\]

\[W(\theta)(1 + P_z^h(\theta)\alpha_B \cos \theta_B + P_x^h(\theta)\alpha_B \sin \theta_B \cos \chi)
\]

\(\theta\)-dependent hadron-side polarization components

\[P_z^h(\theta) = \frac{1}{W(\theta)} \left[ \frac{3}{8} (1 + \cos^2 \theta) H_p + \frac{3}{4} \cos \theta H_U + \frac{3}{4} \sin^2 \theta H_{Lp} + \right.
\]

\[+ \delta_\ell \left( \frac{3}{4} \cos^2 \theta H_{Lp} + \frac{3}{4} \sin^2 \theta H_p + \frac{3}{2} H_{Sp} - 3 \cos \theta H_{Sp} \right) \]

\[P_x^h(\theta) = \frac{1}{W(\theta)} \left[ \mp \frac{3}{2\sqrt{2}} \sin \theta H_{LT} + \frac{3}{4\sqrt{2}} \sin 2\theta H_{LTp} + \right.
\]

\[+ \delta_\ell \left( \frac{3}{2\sqrt{2}} \sin 2\theta H_{LTp} + \frac{3}{\sqrt{2}} \sin \theta H_{STp} \right) \]
The angular decay distribution for the cascade decay
\[ \Lambda_b^0 \rightarrow \Lambda_c^+ (\rightarrow \Lambda_c^0 + \pi^+) + l^- + \bar{\nu}_l \]

Figure: Definition of the polar angles \( \theta, \theta_B \) and the azimuthal angle \( \chi \).
Covariant quark model

- Transition $\Lambda_b = (b[ud])$ to $\Lambda_c = (c[ud])$
- Lagrangian describing the interaction $\Lambda_Q$ baryon with its constituents:
  \[ L^\Lambda_Q_{\text{int}} (x) = g_{\Lambda_Q} \bar{\Lambda}_Q (x) \cdot J^{\Lambda_Q} (x) + g_{\Lambda_Q} \bar{J}_{\Lambda_Q} (x) \cdot \Lambda_Q (x) \]
- Three-quark currents:
  \[ J^{\Lambda_Q}_{\Lambda_Q} (x) = \int dx_1 \int dx_2 \int dx_3 F^{\Lambda_Q} (x; x_1, x_2, x_3) J^{(\Lambda_Q)}_{3q} (x_1, x_2, x_3) \]
  \[ J^{(\Lambda_Q)}_{3q} (x_1, x_2, x_3) = \epsilon^{a_1 a_2 a_3} Q^{a_1} (x_1) u^{a_2} (x_2) C \gamma^5 d^{a_3} (x_3) \]
  \[ \bar{J}^{\Lambda_Q}_{\Lambda_Q} (x) = \int dx_1 \int dx_2 \int dx_3 F^{\Lambda_Q} (x; x_1, x_2, x_3) \bar{J}^{(\Lambda_Q)}_{3q} (x_1, x_2, x_3) \]
  \[ \bar{J}^{(\Lambda_Q)}_{3q} (x_1, x_2, x_3) = \epsilon^{a_1 a_2 a_3} \bar{d}^{a_3} (x_3) \gamma^5 C \bar{u}^{a_2} (x_2) \bar{Q}^{a_1} (x_1) \]
Covariant quark model

• The vertex function $F_{Q\Lambda}$ is chosen to be of the form

$$F_{\Lambda}(x; x_1, x_2, x_3) = \delta^{(4)} \left( x - \sum_{i=1}^{3} w_i x_i \right) \Phi_{\Lambda} \left( \sum_{i<j} (x_i - x_j)^2 \right)$$

where $w_i = m_i/m_1 + m_2 + m_3$.

• We choose a simple Gaussian form for the function $\Phi_{\Lambda}$:

$$\Phi_{\Lambda}(-P^2) = \exp(P^2/\Lambda^2)$$

where the parameter $\Lambda$ characterizes the hadron size.

• The three quark propagator in momentum space

$$S_f = \frac{1}{m_f - k}$$

where $f = u, d, c, b$ denotes the flavor of the freely propagating quark.
Compositeness condition  $Z_H = 0$

- The coupling constant $g_\Lambda$ is determined by the compositeness condition

$$Z_\Lambda = 1 - \Sigma'_\Lambda (m_\Lambda) = 0$$

where $\Sigma'_\Lambda$ is the on-shell derivative of the $\Lambda$-type baryon mass function $\Sigma_\Lambda$, i.e. $\Sigma'_\Lambda = \partial \Sigma_\Lambda / \partial p$.

![Diagram of $\Lambda$-type baryon mass operator]

Figure: $\Lambda_{q_1q_2q_3}$ baryon mass operator.

**Model parameters**

<table>
<thead>
<tr>
<th>$m_u$</th>
<th>$m_s$</th>
<th>$m_c$</th>
<th>$m_b$</th>
<th>$m_\tau$</th>
<th>GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.241</td>
<td>0.428</td>
<td>1.67</td>
<td>5.04</td>
<td>1.7768</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Lambda_{\Lambda_c}$</th>
<th>$\Lambda_{\Lambda_b}$</th>
<th>$M_{\Lambda_c}$</th>
<th>$M_{\Lambda_b}$</th>
<th>$\lambda$</th>
<th>GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.867</td>
<td>0.571</td>
<td>2.2864</td>
<td>5.6195</td>
<td>0.181</td>
<td></td>
</tr>
</tbody>
</table>
**Numerical results**

Helicity structure functions in units of $10^{-15}$ GeV.

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_U$</th>
<th>$\Gamma_L$</th>
<th>$\Gamma_{LT}$</th>
<th>$\Gamma_P$</th>
<th>$\Gamma_{LP}$</th>
<th>$\Gamma_{LTP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>12.4</td>
<td>19.6</td>
<td>-7.73</td>
<td>-7.61</td>
<td>-18.5</td>
<td>-3.50</td>
</tr>
<tr>
<td>$\tau$</td>
<td>3.29</td>
<td>2.90</td>
<td>-2.06</td>
<td>-1.73</td>
<td>-2.46</td>
<td>-0.66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\tilde{\Gamma}_U$</th>
<th>$\tilde{\Gamma}_L$</th>
<th>$\tilde{\Gamma}_S$</th>
<th>$\tilde{\Gamma}_{LT}$</th>
<th>$\tilde{\Gamma}_{SP}$</th>
<th>$\tilde{\Gamma}_{SL}$</th>
<th>$\tilde{\Gamma}_P$</th>
<th>$\tilde{\Gamma}_{LP}$</th>
<th>$\tilde{\Gamma}_{LTP}$</th>
<th>$\tilde{\Gamma}_{STP}$</th>
<th>$\tilde{\Gamma}_{SLP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>0.66</td>
<td>0.63</td>
<td>0.64</td>
<td>-0.41</td>
<td>-0.55</td>
<td>0.55</td>
<td>-0.37</td>
<td>-0.55</td>
<td>-0.14</td>
<td>-0.42</td>
<td>-0.64</td>
</tr>
</tbody>
</table>

The values of the forward-backward asymmetry $\langle A_{FB}^l \rangle$, the convexity parameter $\langle C_F \rangle$, the hadronic $\langle P_{z,x}^h \rangle$ and leptonic $\langle P_{z,x}^l \rangle$ polarization components

<table>
<thead>
<tr>
<th></th>
<th>$\langle A_{FB}^l \rangle$</th>
<th>$\langle C_F \rangle$</th>
<th>$\langle P_{z}^h \rangle$</th>
<th>$\langle P_{x}^h \rangle$</th>
<th>$\langle P_{z}^l \rangle$</th>
<th>$\langle P_{x}^l \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-\nu_e$</td>
<td>0.36</td>
<td>-0.63</td>
<td>-0.82</td>
<td>0.40</td>
<td>-1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\tau^-\nu_\tau$</td>
<td>-0.077</td>
<td>-0.10</td>
<td>-0.72</td>
<td>0.22</td>
<td>-0.32</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Numerical results

The $q^2$-dependence of the partial rates (in units of $10^{-15}$ GeV$^{-1}$)
The $q^2$ - dependence of the lepton-side forward-backward asymmetry $A_{FB}^l(q^2)$ for the $e^-$- (solid) and $\tau^-$ - mode (dashed).

The $q^2$ - dependence of the convexity parameter $C_F(q^2)$ for the $e^-$ - (solid) and $\tau^-$ - mode (dashed).
The $q^2$ - dependence of the longitudinal, transverse and total polarization of the daughter baryon $\Lambda_c$ for the $e^-$- (solid) and $\tau^-$- mode (dashed).
The $q^2$-dependence of the longitudinal, transverse and total polarization for the charged leptons $e^-$ (solid) and $\tau^-$-mode (dashed).
Conclusions

• We have used the helicity formalism to study the angular decay distribution in the semileptonic decay $\Lambda_b^0 \rightarrow \Lambda_c^+ + \tau^- + \bar{\nu}_\tau$, as well as the corresponding cascade decay $\Lambda_b^0 \rightarrow \Lambda_c^0(\rightarrow \Lambda_c^0 + \pi^+) + \tau^- + \bar{\nu}_\tau$.

• We calculate the $\Lambda_b \rightarrow \Lambda_c$ transition form factors in the framework of covariant quark model and use them to evaluate a set of polarization observables.

• All our results have been presented for the two cases: the near mass zero leptons $\ell = e, \mu$ and the massive $\tau$ lepton $\ell = \tau$. 
THANK YOU FOR YOUR ATTENTION!