CPLEAR data on $K^0(\bar{K}^0) \rightarrow \pi^{\pm}e^{\mp}\nu$ and the mass difference $(m_L - m_S)$ determination

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INTRODUCTION

RELATION BETWEEN MASS DIFFERENCES \((m_L - m_S)\) and \((m_2 - m_1)\)

OSCILLATIONS OF NEUTRAL \(K\) MESONS THROUGH \(K_1^0, K_2^0\)

OSCILLATIONS OF NEUTRAL \(K\) MESONS THROUGH \(K_S^0, K_L^0\)

CONCLUSIONS
The neutral $K$-mesons $K^0, \bar{K}^0$, produced in strong and EM interactions, appear with charged $K$-mesons as the iso-doublets $(K^+, K^0)$ and $(K^-, \bar{K}^0)$ in the nonet of pseudoscalar mesons - $\pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, \eta, \eta'$ - characterized by the quantum numbers $I(J^P) = 1/2(0^-)$ and the ”strangenes” $S = +1$ and $S = -1$, respectively.
The single $K^0$ mesons can be produced e.g. in the process
\[ \pi^- p \to K^0 \Lambda^0. \]

However, ”concurrently” $K^0$ and $\bar{K}^0$ are produced by the $\bar{p}$ annihilation at rest in a hydrogen target
\[ \bar{p} p \to K^- \pi^+ K^0 \text{ or } K^+ \pi^- \bar{K}^0 \text{ (CPLEAR at CERN) } \]
each having the branching ratio $2 \times 10^{-3}$.

Then the ”strangenes” $S$ of neutral $K$-mesons can be ”tagged” by measuring the charge sign of the accompanying $K^\pm$ - therefore it is known ”event by event”.

S. Dubnicka, Anna Z. Dubnickova, CPLEAR data on $K^0(\bar{K}^0) \to \pi^\pm e^\mp \nu$ and the mass difference.
Another possibility to "tag" $S$ of produced $K^0$ and $\bar{K}^0$ is an investigation of their subsequent semi-leptonic decays

$$K \rightarrow l \pi \nu \quad (l = e, \mu)$$

by the charge-sign of the e.g. "positron" and "electron" in final state, as

$$K_0 \rightarrow e^+ \pi^- \nu$$

$$\bar{K}_0 \rightarrow e^- \pi^+ \bar{\nu}.$$
The quantum number "strangeness" $S$ is conserved in strong and EM interactions.

The violation of $S$ in weak interaction is responsible not only for decays of $K^0$ and $\bar{K}^0$ mesons, but also gives rise to the so-called "oscillations" of neutral $K$-mesons $K^0 \leftrightarrow \bar{K}^0$ in time.
Both $K^0$ and $\bar{K}^0$ can decay into two pions

$\pi^0\pi^0, \pi^+\pi^-$

and also into three pions

$\pi^0\pi^0\pi^0, \pi^+\pi^-\pi^0,$

whereby the pion system possess well defined "$CP$-parity", +1 and −1, respectively.

However - neither $K^0$ nor $\bar{K}^0$ are eigenstates of $CP$ operator

$CP|K^0\rangle = -|\bar{K}^0\rangle$

$CP|\bar{K}^0\rangle = -|K^0\rangle.
On account of this reason new particles $K^0_1$ and $K^0_2$ have been defined to exist as a superposition of $K^0$ and $\bar{K}^0$

$$|K^0_1\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle)$$

$$|K^0_2\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle)$$

with definite $CP$ parity values

$CP|K^0_1\rangle = +|K^0_1\rangle$

$CP|K^0_2\rangle = -|K^0_2\rangle$.

As a consequence of the latter $K^0_1$ can decay into two pions and $K^0_2$ can decay into three pions.
But - in 1964

*Christenson, Cronin, Fitch and Turlay, Phys. Rev Lett. 13 (1964) 138*

have revealed decay $K_2^0 \rightarrow \pi^+\pi^-, \pi^0\pi^0$ with some small probability - violating $CP$-invariance.

As a consequence another two neutral $K$-mesons, $K_S^0$ and $K_L^0$ to be a linear combinations of $K_1^0$ and $K_2^0$, have been introduced to exist

\[
\begin{align*}
|K_S^0\rangle &= \frac{1}{\sqrt{1 + |\varepsilon|^2}} (|K_1^0\rangle + \varepsilon |K_2^0\rangle) \\
|K_L^0\rangle &= \frac{1}{\sqrt{1 + |\varepsilon|^2}} (\varepsilon |K_1^0\rangle + |K_2^0\rangle)
\end{align*}
\]

where $\varepsilon$ is a complex $CP$-violation parameter, $|\varepsilon| = 2, 3.10^{-3}$ and the $CP$-violation phase $\Phi = 43, 5^\circ$. 

\[\text{CLEAR data on } K^0(\bar{K}^0) \rightarrow \pi^\pm e^\mp \nu \text{ and the mass difference}\]
Next $K^0 \leftrightarrow \bar{K}^0$ "oscillations" in vacuum are investigated.

First, oscillations through $K_1^0$ and $K_2^0$ mesons, when $CP$-invariance is considered.

Then, oscillations through $K_S^0$ and $K_L^0$ mesons, considering the $CP$-violation.

**Note:**
However, according to our knowledge there are no experimental results about direct measurements of the $K^0 \leftrightarrow \bar{K}^0$ "oscillations" in vacuum.

Only measurements of the semi-leptonic decays of neutral $K$-mesons

\[
K^0 \rightarrow \pi^- e^+ \nu \\
\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}
\]

are carried out and the data on the so-called asymmetry (see Fig.), extracting four decay rates $R$ as a function of the decay eigentime $\tau$.
\[
R_+(\tau) = R(K^0_{t=0} \rightarrow \pi^- e^+ \nu_{t=\tau}) \\
\bar{R}_-(\tau) = R(\bar{K}^0_{t=0} \rightarrow \pi^+ e^- \bar{\nu}_{t=\tau}) \\
R_-(\tau) = R(K^0_{t=0} \rightarrow \pi^+ e^- \bar{\nu}_{t=\tau}) \\
\bar{R}_+(\tau) = R(\bar{K}^0_{t=0} \rightarrow \pi^- e^+ \nu_{t=\tau})
\]

\[
A_{exp}(\tau) = \frac{[R_+(\tau) + \bar{R}_-(\tau)] - [\bar{R}_+(\tau) + R_-(\tau)]}{[R_+(\tau) + \bar{R}_-(\tau)] + [\bar{R}_+(\tau) + R_-(\tau)]}
\]

have been obtained


from CPLEAR at CERN.
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**Figure:** The data on ASYMMETRY to be obtained by CPLEAR Coll.
But, before the latter, a relation between the mass difference of $K_1^0$ and $K_2^0$, $(m_2 - m_1)$ and the mass difference of $K_S^0$ and $K_L^0$, $(m_L - m_S)$ is derived in order to demonstrate that they are not identical.
The complex masses of the $K_1^0$ and $K_2^0$ mesons can be defined as matrix elements from the weak Hamiltonian between $K_1^0$ and $K_2^0$ meson state vectors.

If one substitutes instead of the latter expressions to be expressed through $|K^0\rangle$ and $|\bar{K}^0\rangle$, one gets relations

$$m_1 - i\frac{\Gamma_1}{2} = \langle K_1^0 | H^w | K_1^0 \rangle = \frac{1}{2}[\langle K^0 | H^w | K^0 \rangle - \langle \bar{K}^0 | H^w | K^0 \rangle - \langle K^0 | H^w | \bar{K}^0 \rangle + \langle \bar{K}^0 | H^w | \bar{K}^0 \rangle]$$

$$m_2 - i\frac{\Gamma_2}{2} = \langle K_2^0 | H^w | K_2^0 \rangle = \frac{1}{2}[\langle K^0 | H^w | K^0 \rangle + \langle \bar{K}^0 | H^w | K^0 \rangle + \langle K^0 | H^w | \bar{K}^0 \rangle + \langle \bar{K}^0 | H^w | \bar{K}^0 \rangle]$$
from where for the mass difference of $K_2^0$ and $K_1^0$ mesons one obtains

$$(m_2 - m_1) = \text{Re}\langle \bar{K}^0 \mid H^w \mid K^0 \rangle + \text{Re}\langle K^0 \mid H^w \mid \bar{K}^0 \rangle$$
Now, similarly

\[
m_s - i \frac{\Gamma_s}{2} = \langle K_s^0 \mid H^w \mid K_s^0 \rangle = \frac{1}{2(1 + |\epsilon|^2)} \times \\
\left[ (1 + \epsilon^*)(1 + \epsilon)\langle K^0 \mid H^w \mid K^0 \rangle - \\
- (1 - \epsilon^*)(1 + \epsilon)\langle \tilde{K}^0 \mid H^w \mid K^0 \rangle - \\
- (1 + \epsilon^*)(1 - \epsilon)\langle K^0 \mid H^w \mid \tilde{K}^0 \rangle + \\
+ (1 - \epsilon^*)(1 - \epsilon)\langle \tilde{K}^0 \mid H^w \mid \tilde{K}^0 \rangle \right]
\]
\[ m_L - i \frac{\Gamma_L}{2} = \langle K^0_L | H^w | K^0_L \rangle = \frac{1}{2(1 + |\varepsilon|^2)} \times \]

\[
\left[ (1 + \varepsilon^*)(1 + \varepsilon) \langle K^0 | H^w | K^0 \rangle + \\
+ (1 - \varepsilon^*)(1 + \varepsilon) \langle \bar{K}^0 | H^w | K^0 \rangle + \\
+ (1 + \varepsilon^*)(1 - \varepsilon) \langle K^0 | H^w | \bar{K}^0 \rangle + \\
+ (1 - \varepsilon^*)(1 - \varepsilon) \langle \bar{K}^0 | H^w | \bar{K}^0 \rangle \right]
\]
and for the mass difference of $K_L^0$ and $K_S^0$ mesons one obtains the relation

$$\left( m_L - m_S \right) = \frac{(1-|\varepsilon|)^2}{(1+|\varepsilon|)^2} (m_2 - m_1) +$$

$$+ \frac{2Im\varepsilon}{(1+|\varepsilon|^2)} \left[ Im\langle K^0 | H^w | \bar{K}^0 \rangle - Im\langle \bar{K}^0 | H^w | K^0 \rangle \right]$$

from which one can see immediately that both mass differences are not identical.
Now one can write relations

\[ |K^0\rangle = \frac{1}{\sqrt{2}}(|K_1^0\rangle + |K_2^0\rangle) \]

\[ |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}(-|K_1^0\rangle + |K_2^0\rangle), \]

which are just the **inverse transformations** to those by means of which the existence of \(|K_1^0\rangle\) and \(|K_2^0\rangle\) has been introduced previously.
The time dependence of state vectors of $K_1^0, K_2^0$ is

$$|K_1^0(t)\rangle = e^{im_1 t - \Gamma_1/2t} |K_1^0(0)\rangle$$
$$|K_2^0(t)\rangle = e^{im_2 t - \Gamma_2/2t} |K_2^0(0)\rangle$$

with $m_1, m_2$ and $\Gamma_1, \Gamma_2$ the masses and widths of $K_1^0, K_2^0$, respectively.
Then, for the state vectors \( |K^0(t)\rangle\), \( |\bar{K}^0(t)\rangle\) one can write expressions

\[
|K^0(t)\rangle = \frac{1}{2} \left[ e^{-im_1 t - \Gamma_1/2t} + e^{-im_2 t - \Gamma_2/2t} \right] \cdot |K^0(0)\rangle + \frac{1}{2} \left[ e^{-im_2 t - \Gamma_2/2t} - e^{-im_1 t - \Gamma_1/2t} \right] \cdot |\bar{K}^0(0)\rangle
\]

\[
|\bar{K}^0(t)\rangle = \frac{1}{2} \left[ e^{-im_2 t - \Gamma_2/2t} - e^{-im_1 t - \Gamma_1/2t} \right] \cdot |K^0(0)\rangle + \frac{1}{2} \left[ e^{-im_1 t - \Gamma_1/2t} + e^{-im_2 t - \Gamma_2/2t} \right] \cdot |\bar{K}^0(0)\rangle
\]

to be ready for calculations of the \( K^0 \leftrightarrow \bar{K}^0 \) oscillations.
In order to find an explicit form of the **theoretical asymmetry** \( A_{th}(\tau) \) one has to calculate probabilities of the following transitions:

\[
P(K^0(0) \rightarrow \bar{K}^0(t)), \quad P(\bar{K}^0(0) \rightarrow K^0(t)), \quad P(K^0(0) \rightarrow K^0(t)) \quad \text{and} \quad P(\bar{K}^0(0) \rightarrow \bar{K}^0(t)).
\]
The probability - that the \( K^0 \) meson produced at the moment \( t = 0 \) will be at the moment \( t \neq 0 \) in the state of \( \bar{K}^0 \) meson, is given by the absolute value squared of the product 
\[
\langle K^0(0) \parallel \bar{K}^0(t) \rangle.
\]
Similarly the reversed probability, whereby the orthogonality of \( K^0(0) \) and \( \bar{K}^0(0) \) states is exploited. The result is

\[
P(K^0(0) \rightarrow \bar{K}^0(t)) \equiv P(\bar{K}^0(0) \rightarrow K^0(t)) = \frac{1}{4} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2\cos[(m_2 - m_1)t]e^{-\frac{(\Gamma_1 + \Gamma_2)}{2}t}]
\]
as we consider the \textit{CP-invariance} which creates \textit{T-invariance} because of the \textit{CPT} conservation.
One has to calculate also $P(K^0(0) \rightarrow K^0(t))$ and $P(\bar{K}^0(0) \rightarrow \bar{K}^0(t))$ transitions in a similar way, which are taking the following forms

$$P(K^0(0) \rightarrow K^0(t)) \equiv P(\bar{K}^0(0) \rightarrow \bar{K}^0(t)) = \frac{1}{4} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2\cos[(m_2 - m_1)t]e^{-\frac{(\Gamma_1 + \Gamma_2)}{2}t}].$$
Substituting all these probabilities of the corresponding transitions into the theoretical Asymmetry

\[
A_{th}(t) = \frac{[P_{K^0(0)\to K^0(t)} + P_{\bar{K}^0(0)\to \bar{K}^0(t)}] - [P_{\bar{K}^0(0)\to K^0(t)} + P_{K^0(0)\to \bar{K}^0(t)}]}{[P_{K^0(0)\to K^0(t)} + P_{\bar{K}^0(0)\to \bar{K}^0(t)}] + [P_{\bar{K}^0(0)\to K^0(t)} + P_{K^0(0)\to \bar{K}^0(t)}]}
\]

finally, the following three-parametric expression

\[
A_{th}(t) = \frac{2 \cos[(m_2 - m_1)t]e^{-(\Gamma_1+\Gamma_2)/2}t}{e^{-\Gamma_1t} + e^{-\Gamma_2t}}
\]

is found, in which, however, the time \( t \) has to be redefined \( t' = \frac{t}{\tau_1} \).
The neutral $K_S^0$ and $K_L^0$ mesons have been introduced by the relations

\[
|K_S^0\rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}}(|K_1^0\rangle + \epsilon |K_2^0\rangle)
\]

\[
|K_L^0\rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}}(\epsilon |K_1^0\rangle + |K_2^0\rangle)
\]

where $\epsilon$ is a complex $CP$-violation parameter, $|\epsilon| = 2, 3.10^{-3}$ and the $CP$-violation phase $\Phi = 43, 5^\circ$.

If for $K_1^0$ and $K_2^0$ the previous relations

\[
|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)
\]

\[
|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)
\]
are substituted, then one obtains $K^0_S$, $K^0_L$ to be expressed through $K^0$ and $\bar{K}^0$ as follows

$$|K^0_S\rangle = \frac{1}{\sqrt{2}\sqrt{1 + |\varepsilon|^2}}[(1 + \varepsilon)|K^0\rangle - (1 - \varepsilon)|\bar{K}^0\rangle]$$

$$|K^0_L\rangle = \frac{1}{\sqrt{2}\sqrt{1 + |\varepsilon|^2}}[(1 + \varepsilon)|K^0\rangle + (1 - \varepsilon)|\bar{K}^0\rangle]$$

and the inverse relations to them are
\[ | K^0 \rangle = \frac{\sqrt{1 + |\varepsilon|^2}}{\sqrt{2(1 + \varepsilon)}} [ | K^0_S \rangle + | K^0_L \rangle ] \]

\[ | \bar{K}^0 \rangle = \frac{\sqrt{1 + |\varepsilon|^2}}{\sqrt{2(1 - \varepsilon)}} [- | K^0_S \rangle + | K^0_L \rangle ] . \]

The **time evolution** of state vectors \( | K^0_S \rangle \) and \( | K^0_L \rangle \) is given by the expressions

\[ | K^0_S(t) \rangle = e^{-i m_S t - \Gamma_S/2t} | K^0_S(0) \rangle \]
\[ | K^0_L(t) \rangle = e^{-i m_L t - \Gamma_L/2t} | K^0_L(0) \rangle \]

with \( m_S, m_L \) and \( \Gamma_S, \Gamma_L \) the masses and widths of \( K^0_S, K^0_L \), respectively, and finally for \( | K^0(t) \rangle \) and \( | \bar{K}^0(t) \rangle \) one can then write
\[ | K^0(t) \rangle = \frac{1}{2} \left[ (e^{-im_st - \Gamma_s/2t} + e^{-imLt - \Gamma_L/2t}) | K^0(0) \rangle + \frac{(1 - \varepsilon)}{(1 + \varepsilon)}(-e^{-im_st - \Gamma_s/2t} + e^{-imLt - \Gamma_L/2t}) | \bar{K}^0(0) \rangle \right] \]

\[ | \bar{K}^0(t) \rangle = \frac{1}{2} \left[ (1 + \varepsilon) \left[ -e^{-im_st - \Gamma_s/2t} + e^{-imLt - \Gamma_L/2t} \right] | K^0(0) \rangle + \frac{(1 + \varepsilon)}{(1 - \varepsilon)} \left[ e^{-imSt - \Gamma_s/2t} + e^{-imLt - \Gamma_L/2t} \right] | \bar{K}^0(0) \rangle \right]. \]
The probability - that the $K^0$ meson produced at the moment $t = 0$ will be at the moment $t \neq 0$ in the state of $\bar{K}^0$ meson, is given by the absolute value squared of the product $\langle K^0(0) | \bar{K}^0(t) \rangle$, i.e.

$$P(K^0(0) \to \bar{K}^0(t)) = \frac{1}{4} \frac{(1 + |\varepsilon|^2 + 2Re\varepsilon)}{(1 + |\varepsilon|^2 - 2Re\varepsilon)} \left[ e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\cos[(m_L - m_S)t]e^{-\frac{(\Gamma_S + \Gamma_L)}{2}t} \right],$$

whereby the orthogonality of $|K^0(0)\rangle$ and $|\bar{K}^0(0)\rangle$ states is exploited.
Similarly the probability of inverse transition

\[ P(\bar{K}^0(0) \rightarrow K^0(t)) = \]
\[ \frac{1}{4} \frac{(1 + |\varepsilon|^2 - 2Re\varepsilon)}{(1 + |\varepsilon|^2 + 2Re\varepsilon)} \left[ e^{-\Gamma_{st}} + e^{-\Gamma_{Lt}} - 2\cos[(m_L - m_S)t]e^{-\frac{\Gamma_{S} + \Gamma_{L}}{2}}t \right]. \]

One can see immediately that

[\[ P(K^0(0) \rightarrow \bar{K}^0(t)) \neq P(\bar{K}^0(0) \rightarrow K^0(t)) \]]

as we consider now CP-violation.
In order to calculate the **theoretical ASYMMETRY** one has to calculate also \( P(K^0(0) \rightarrow K^0(t)) \) and \( P(\bar{K}^0(0) \rightarrow \bar{K}^0(t)) \) in a similar way. They are

\[
P(K^0(0) \rightarrow K^0(t)) \equiv P(\bar{K}^0(0) \rightarrow \bar{K}^0(t)) = \\
\frac{1}{4} [e^{-\Gamma_{st} t} + e^{-\Gamma_{Lt} t} + 2\cos[(m_L - m_S)t]e^{-\frac{(\Gamma_S + \Gamma_L)}{2} t}].
\]

Substituting all these probabilities of the corresponding transitions into the theoretical Asymmetry

\[
A_{th}(t) = \frac{[P_{K^0(0) \rightarrow K^0(t)} + P_{\bar{K}^0(0) \rightarrow \bar{K}^0(t)}] - [P_{\bar{K}^0(0) \rightarrow K^0(t)} + P_{K^0(0) \rightarrow \bar{K}^0(t)}]}{[P_{K^0(0) \rightarrow K^0(t)} + P_{\bar{K}^0(0) \rightarrow \bar{K}^0(t)}] + [P_{\bar{K}^0(0) \rightarrow K^0(t)} + P_{K^0(0) \rightarrow \bar{K}^0(t)}]}
\]

now the following five parametric expression is found
\[ A_{th}(t) = \frac{2\cos[(m_L - m_S)t]e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}} - \frac{4Re^2\varepsilon}{(1+|\varepsilon|^2)^2}(e^{-\Gamma_ST} + e^{-\Gamma_LT})}{(e^{-\Gamma_ST} + e^{-\Gamma_LT}) - \frac{4Re^2\varepsilon}{(1+|\varepsilon|^2)^2}2\cos[(m_L - m_S)t]e^{-\frac{(\Gamma_S + \Gamma_L)t}{2}}} , \]

however, different from the previous relation with characteristics of \( K_1^0, K_2^0 \).

Since experimentally the ASYMMETRY is measured in the mean life time \( \tau \) of \( K_S^0 \), the time \( t \) is again redefined

\[ t' = \frac{t}{\tau_S}.\tau_S . \]
Despite of these differences in $A_{th}(t)$, the authors of the CPLEAR Coll. paper


for a determination of mass differences $K_L - K_S$ have used the $A_{th}(t)$ formula to be derived by considerations of $K^0 \leftrightarrow \bar{K}^0$ "oscillations" through $K_1^0, K_2^0$.

Consequences:
Taking into account results of our presentation, they have measured actually the mass difference of $K_2 - K_1$ mesons and in no case $K_L - K_S$ one as they are declaring.